Exercise 5

Aim of this exercise is to model a hot air balloon and to simulate it. Later, we will formulate a parameter estimation problem with it. Part of the work was done in the lecture already, so we can start with a nonlinear ordinary differential equation (ODE) of the hot air balloon.

Model the system

1. Write a MATLAB function $[\dot{x}]=\text{odefunballoon}(t,x)$ of the hot air balloon system dynamics in the state-space form (to be simulated with ode45)

$$\dot{x} = f(t, x).$$

Here, the state is given by

$$x = \begin{bmatrix} y \\ v \\ T \end{bmatrix}$$

and the (time invariant) model equations by

$$f(t, x) = \begin{bmatrix} v \\ -\beta v - g + \frac{F_B(y)}{m(T)} \\ u - k_2 (T - T_0) / C \end{bmatrix}$$

where

- $y$ [m] is the height
- $v$ [m/s] is the vertical velocity
- $T$ [K] is the temperature of the air in the balloon
- $T_0$ [K] is the outside temperature
- $u$ [W] is the heating power of the burner
- $g = 9.81$ m/s$^2$ is the earth’s gravitational constant
- $F_B(y) = g V \frac{p(y)}{\rho}$ [N] is the buoyancy force
- $p(y) = c_1 - c_2 y$ [Pa] is the pressure (with the pressure assumed to decay linearly).
- $T_0$ [K] is the outside temperature (assumed constant)
- $V$ [m$^3$] is the volume of the balloon (assumed constant)
- $C = C_A V$ [J/K] is the heat capacity of the air in the balloon (assumed constant, product of volumetric heat capacity $C_A$ and volume $V$)
- $k$ [J/K/kg] is the gas constant for air relative to the mass, such that the relation $pV = k m T$ holds.
- $k_2 = A k_3$ [W/K] is the heat loss factor of the balloon, the product of the balloon surface $A$ and the coefficient $k_3$ [W/K/m$^2$] for heat loss per area and Kelvin
- $m(T) = m_0 + m_A(T)$ [kg] is the total mass of the balloon, the combination of the fixed mass $m_0$ of the solid components and the mass of the air $m_A(T) = V \frac{p(y)}{\rho}$
- $\beta$ is a damping coefficient that corresponds to Stokes-type air friction

Based on physical intuition and internet search about balloons, the atmosphere, etc, we choose numerical values for the unknown constants as follows:

- $k_2 = 1200$ W K$^{-1}$
\[ V = 4000 \text{ m}^3 \]
\[ C_A = 1200 \text{ J K}^{-1} \text{ m}^{-3} \]
\[ k = 287 \text{ J K}^{-1} \text{ kg}^{-1} \]
\[ T_0 = 290 \text{ K} \]
\[ c_2 = 12 \text{ Pa m}^{-1} \]
\[ c_1 = 1.013 \cdot 10^5 \text{ Pa} \]
\[ m_0 = 500 \text{ kg} \]
\[ \beta = 0.02 \text{ s}^{-1} \]

In this sheet, we just assume a constant heating power

\[ u = 60000 \text{ W} \]

Note that the input \( t \) (time) is required in the function \( f(t, x) \) because the ODE solver expects it, but that \( t \) does not enter any of the model equations because we have a time independent model.

**Simulate the system**

2. Simulate the balloon for a time of 3600 seconds, using the MATLAB routine \texttt{ode45}. Start the balloon with \( y = 100 \text{ m}, v = 0, T = 390 \text{ K} \).

3. Plot the trajectories for the states.

4. Now simulate with a fixed nonzero control input for a long time. Where does the balloon end?