OPEN DYNAMICAL SYSTEMS

THEIR AIMS and THEIR ORIGINS

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K.U. Leuven, Belgium

VII Antonio Ruberti Lecture **Rome, May 28, 2007**

Antonio Ruberti

Today's area: System theory & control

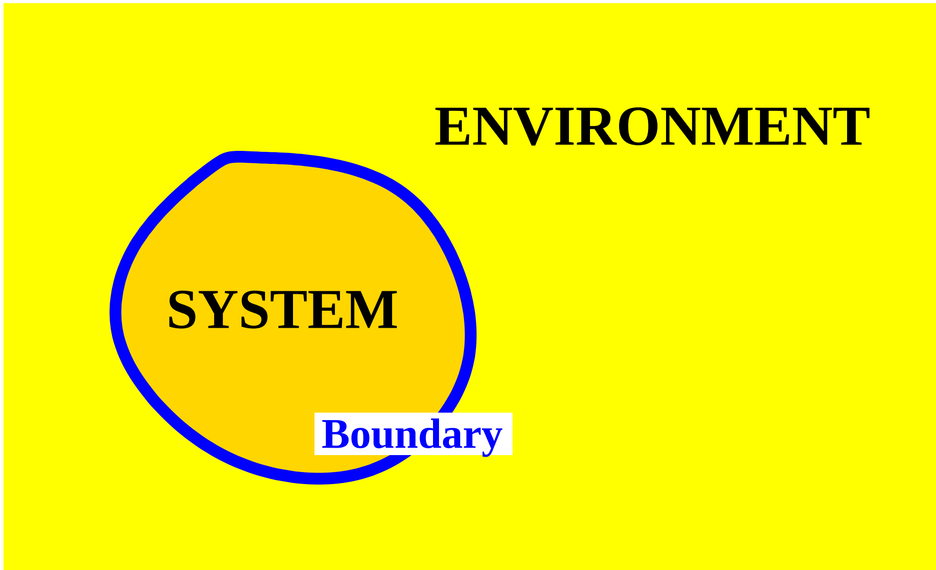
Open and Connected

The central tenets of the field of systems and control:

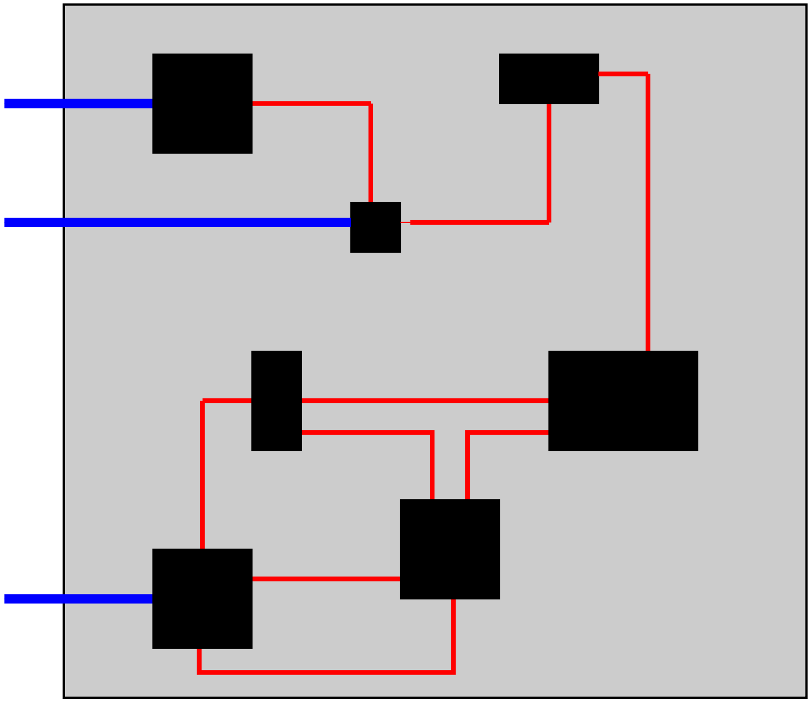
(Typical, generic) systems are open and an interconnection of subsystems.

Synthesis of systems consists of interconnecting subsystems

Open



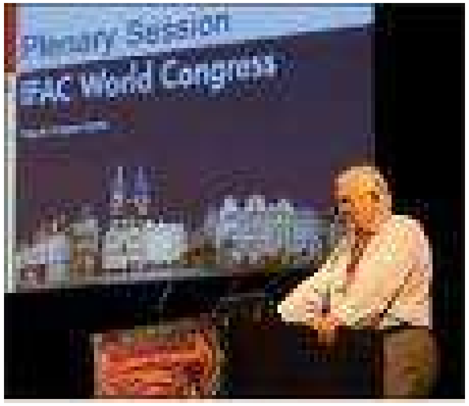
Connected



Architecture with subsystems

Theme

1. **Get the physics right**
2. **The rest is mathematics**



R.E. Kalman, Opening lecture
IFAC World Congress
Prague, July 4, 2005

Theme

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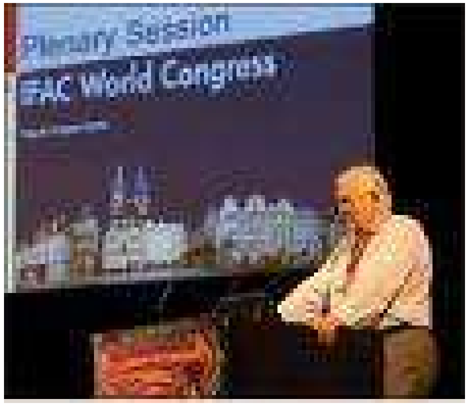

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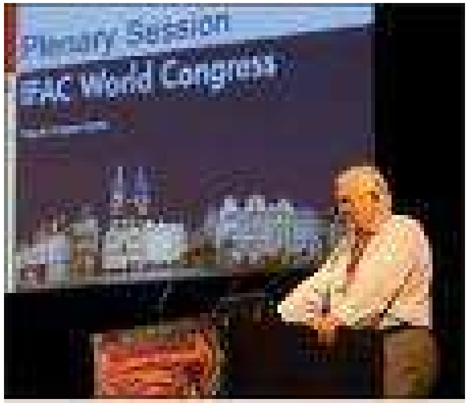
Prima la fisica, poi la matematica

Theme

1. Get the physics right
2. The rest is mathematics



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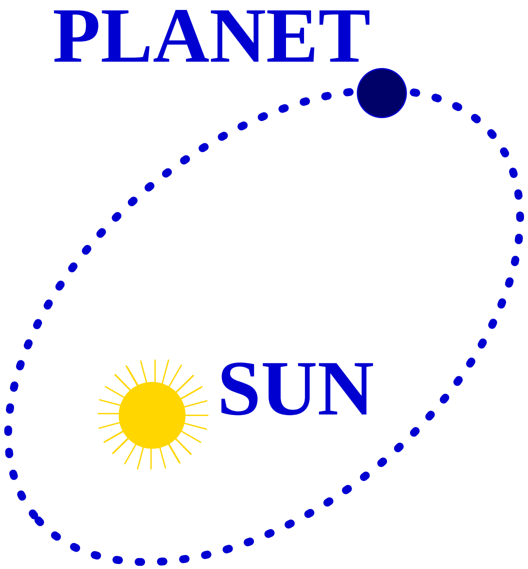
Mathematical analysis, simulation, prediction, work well only if 'axiomatics' is sound, faithful translation of 'reality'...

How it all began ...

How, for heaven's sake, does it move?

Kepler's laws



Johannes Kepler (1571-1630)

Kepler's laws:

Ellipse, sun in focus;
= areas in = times;
(period)^2 $\cong$ (diameter)^3

The equation of the planet

Consequence:

acceleration = function of position and velocity

$$\frac{d^2}{dt^2}w(t) = A(w(t), \frac{d}{dt}w(t))$$

$\rightsquigarrow$ **via calculus and calculation**

$$\frac{d^2}{dt^2}w(t) + \frac{1_{w(t)}}{|w(t)|^2} = 0$$

Isaac Newton (1643-1727)

The equation of the planet

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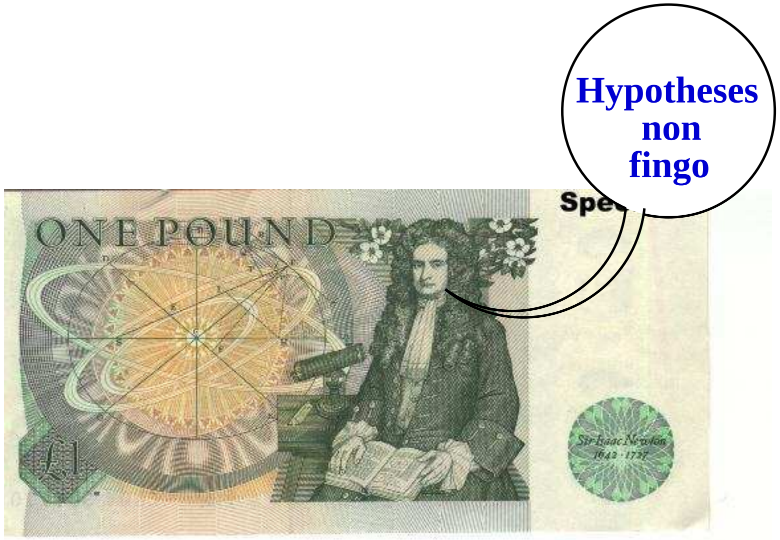


Isaac Newton (1643-1727)

Newton's laws

2-nd law $F'(t) = m\frac{d^2}{dt^2}w(t)$

gravity $F''(t) = m\frac{1_{w(t)}}{|w(t)|^2}$

3-rd law $F'(t) + F''(t) = 0$

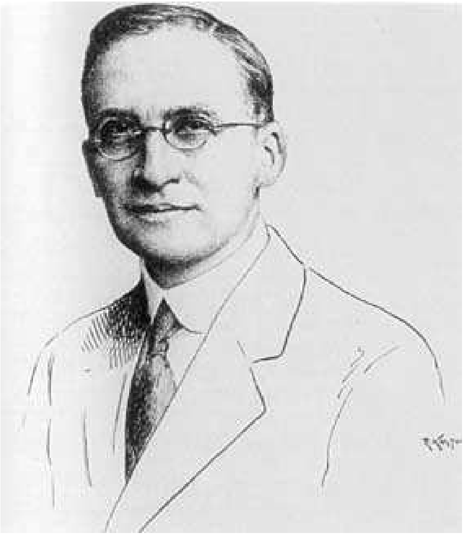
$\Downarrow$

$$\frac{d^2}{dt^2}w(t) + \frac{1_{w(t)}}{|w(t)|^2} = 0$$

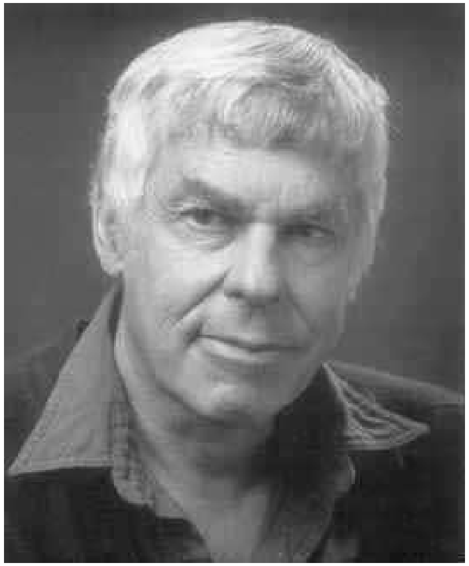
The paradigm of *closed* systems

‘Axiomatization’

Henri Poincaré (1854-1912)



George Birkhoff (1884-1944)



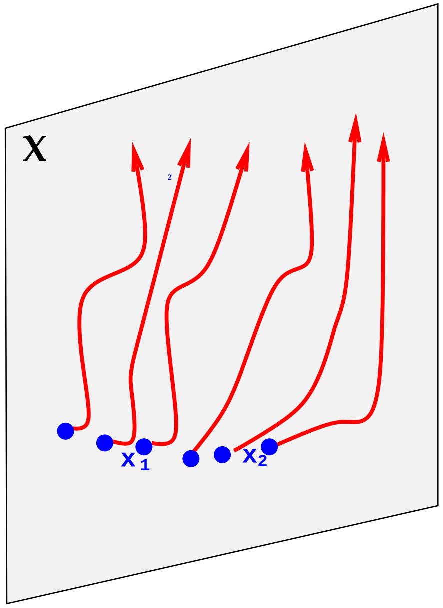
Stephen Smale (1930-)

‘Axiomatization’

A *dynamical system* is defined by

a state space X and

a state transition function



$\phi(t, \mathrm{x})$ = state at time t starting from state x

Dynamics = a set of variables that evolves autonomously in time.

This framework of closed systems

is universally used for dynamics

in mathematics and physics

‘Axiomatization’

A *dynamical system* is defined by

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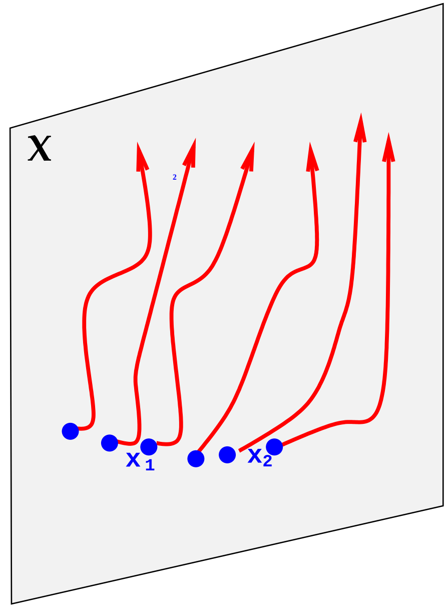
$\phi(t, \mathrm{x})$ **= state at time t starting from state x**



Simulation scenario

= set initial conditions, run model

‘Axiomatization’

A *dynamical system* is defined by

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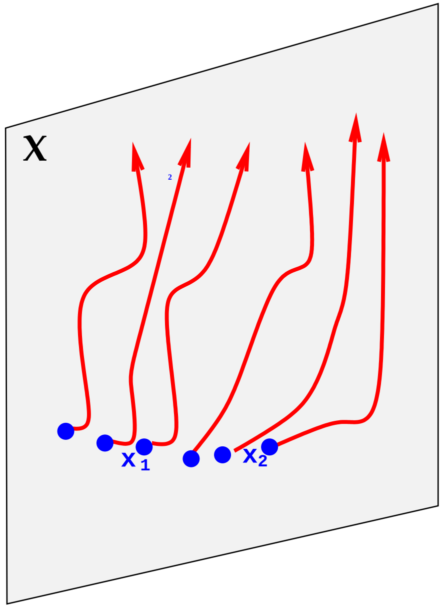

How could they forget Newton’s 2nd law,

about Maxwell’s eq’ns,

about thermodynamics,

about tearing & zooming & linking ...?

Newton's laws

2-nd law $F'(t) = m\frac{d^2}{dt^2}w(t)$

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OPEN SYSTEMS

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CLOSED SYSTEM

$$\frac{d^2}{dt^2}w(t) + \frac{1_{w(t)}}{|w(t)|^2} = 0$$

'Axiomatization'

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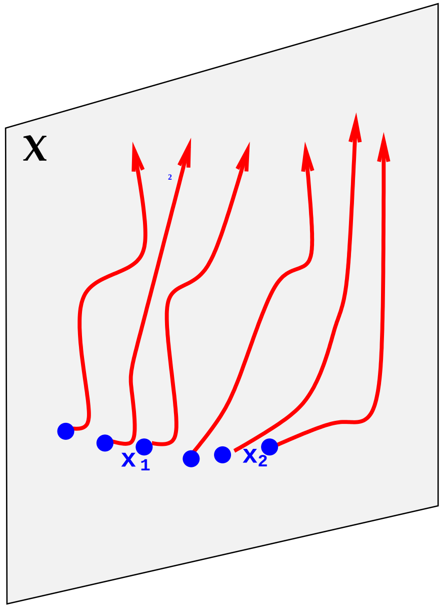


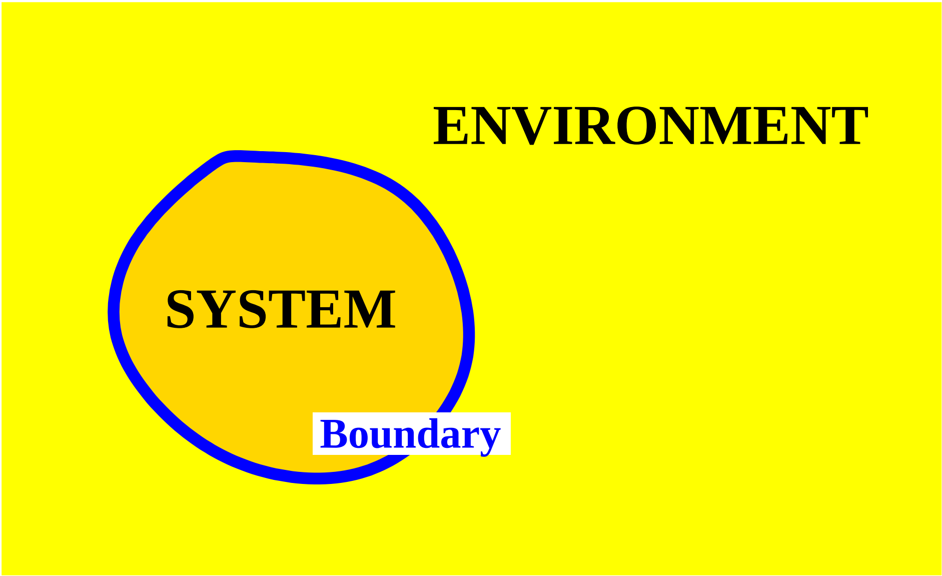
Dynamics = a set of variables that evolves autonomously in time.

How could they forget Newton's 2$^{\text{nd}}$ law,
about Maxwell's eq'ns,
about thermodynamics,
about tearing & zooming & linking ...?

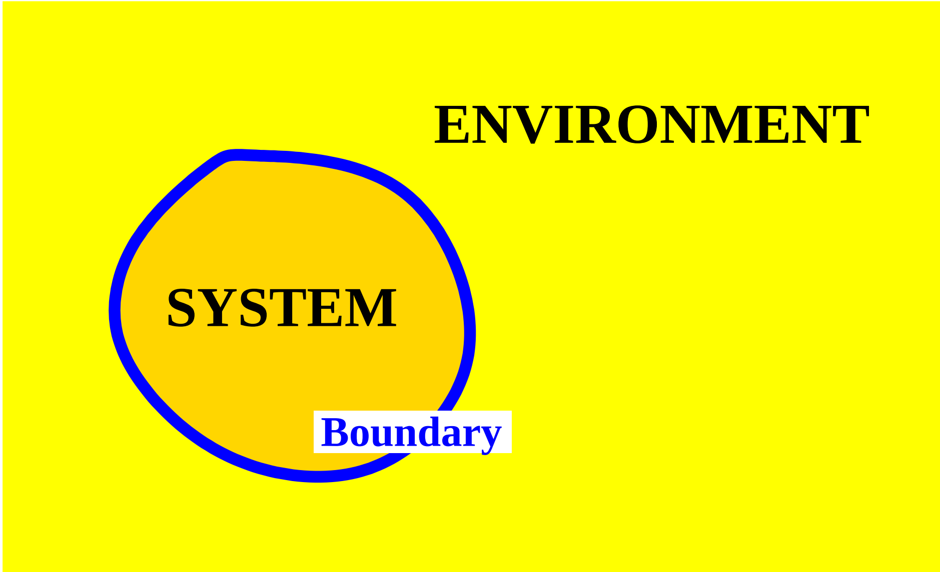
‘Axiomatization’

Reply: assume ‘fixed boundary conditions’

‘fixed’ = ‘known’



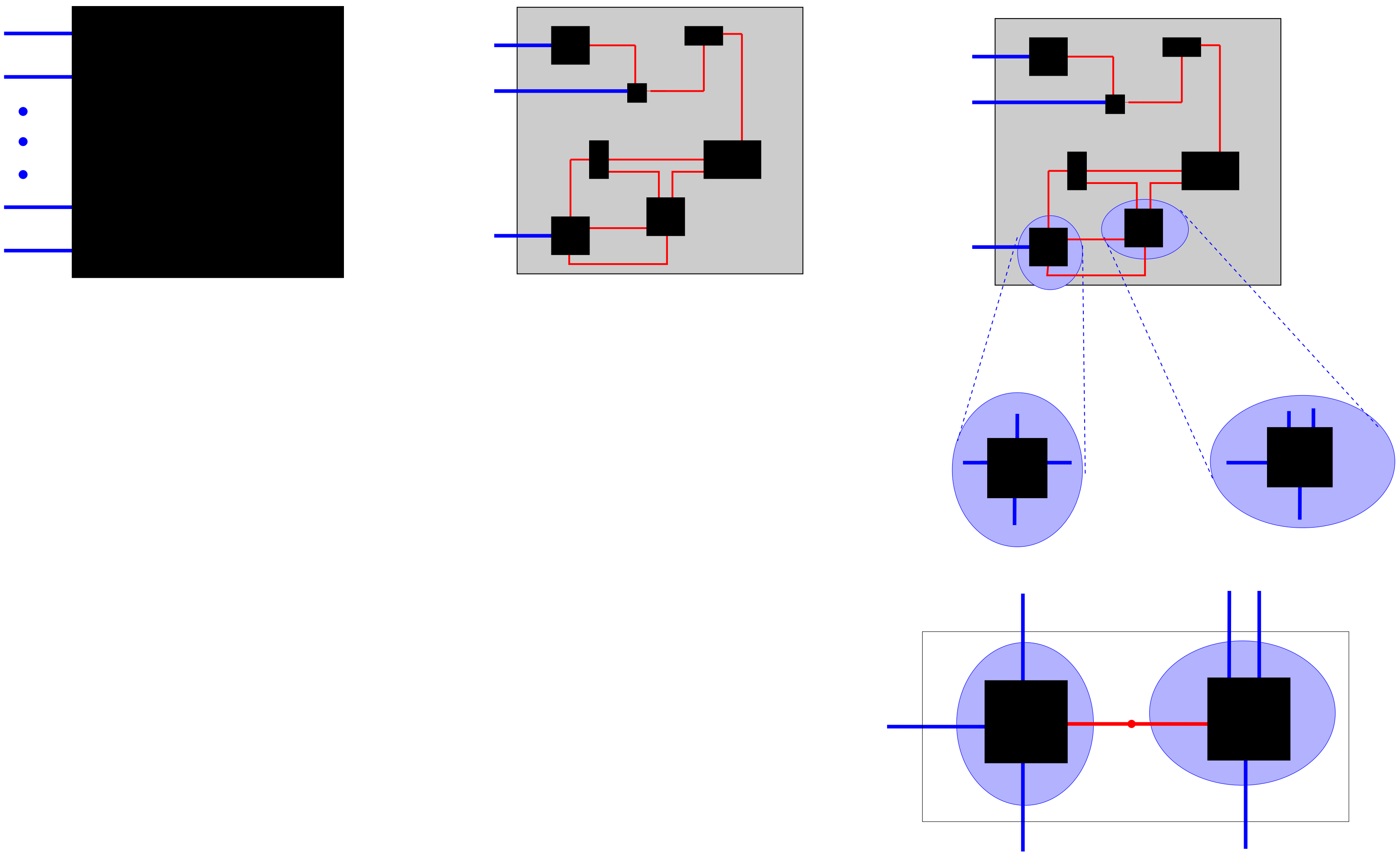
‘Axiomatization’



$\rightsquigarrow$ **to model a system,**
we have to model also the environment!

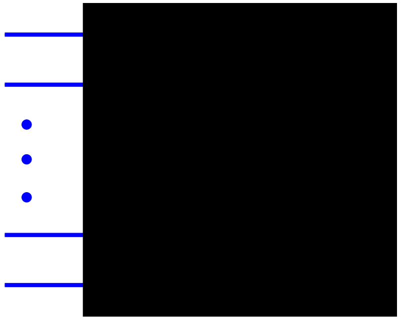
Chaos theory, cellular automata, sync, etc.,
function in this framework ...

Tearing-Zooming-Linking

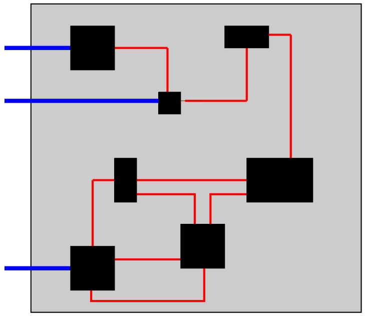


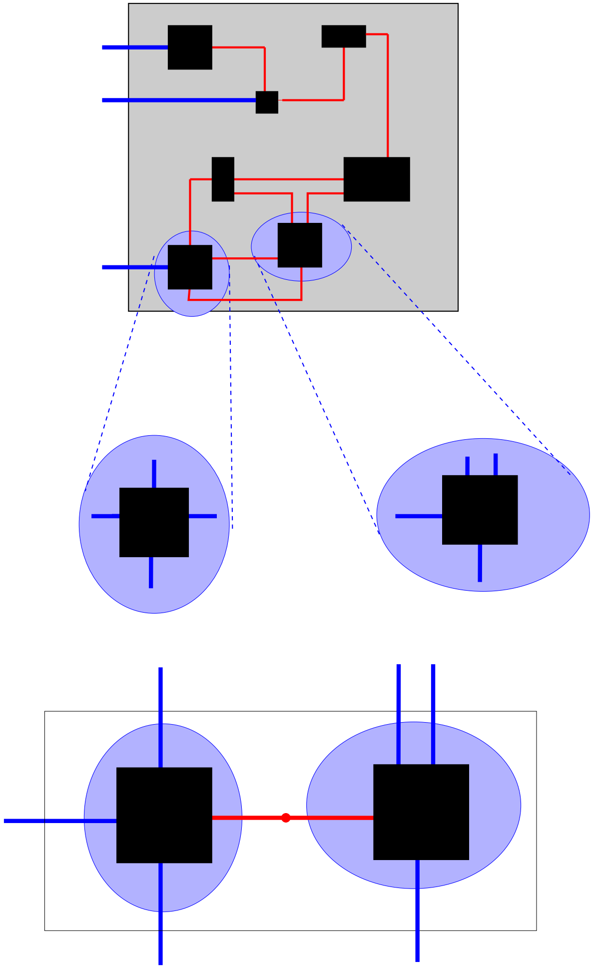
Tearing-Zooming-Linking

Black box

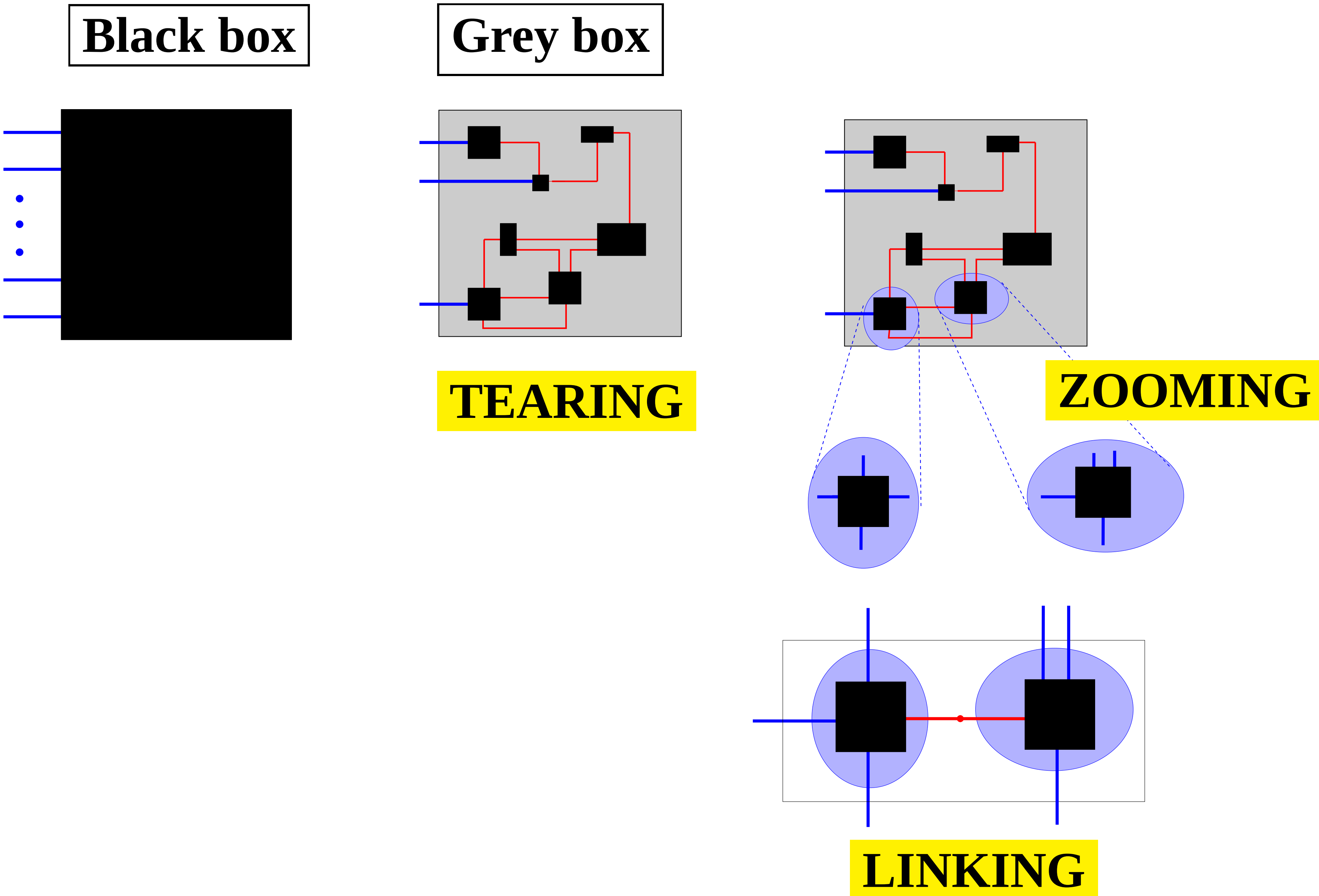


Grey box





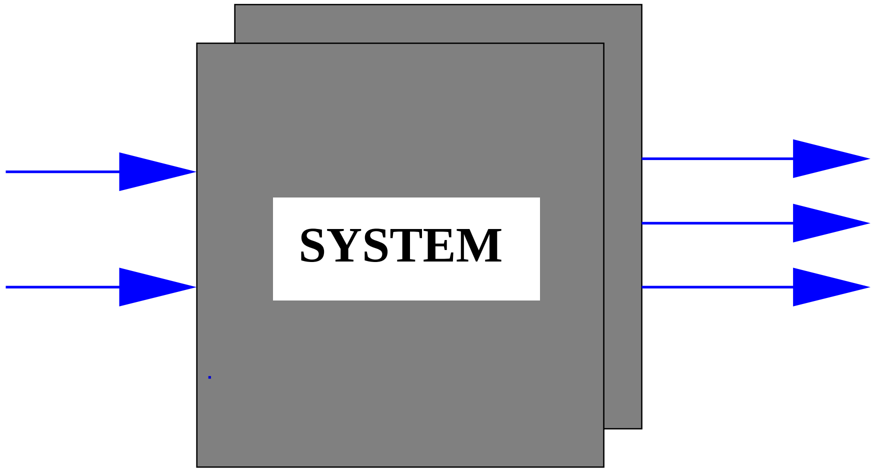
Tearing-Zooming-Linking



Meanwhile, in engineering, ...

Input/output systems

stimulus



response

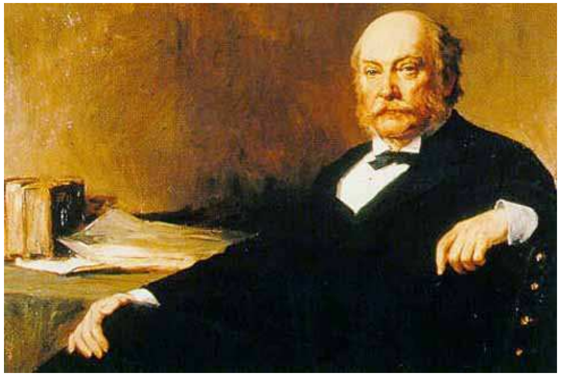
cause
input

effect
output

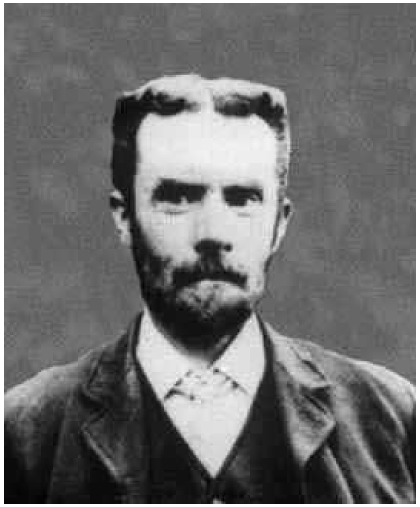
action

reaction

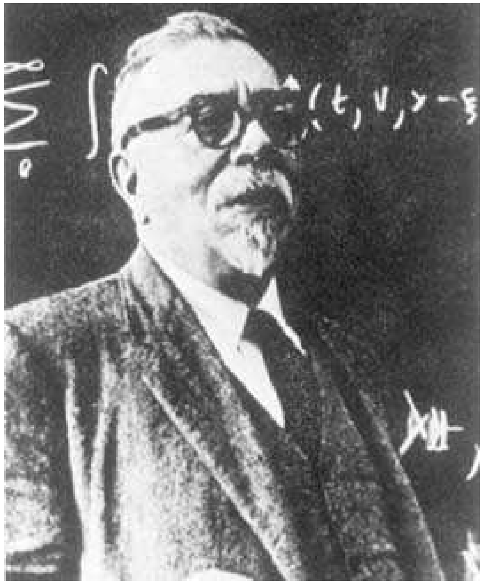
The originators



Lord Rayleigh (1842-1919)



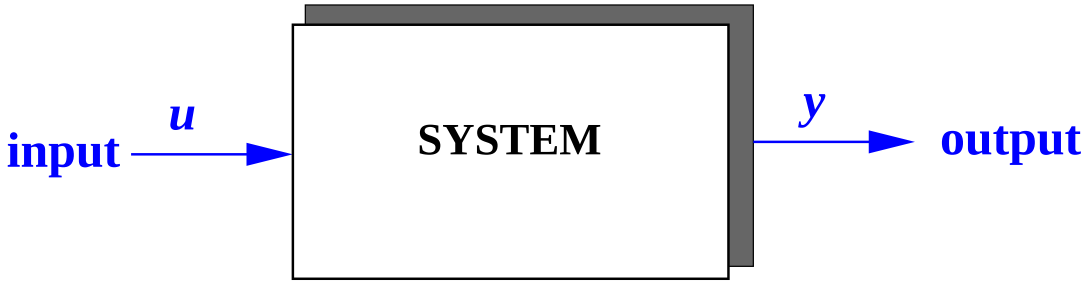
Oliver Heaviside (1850-1925)



Norbert Wiener (1894-1964)

and the many electrical circuit theorists ...

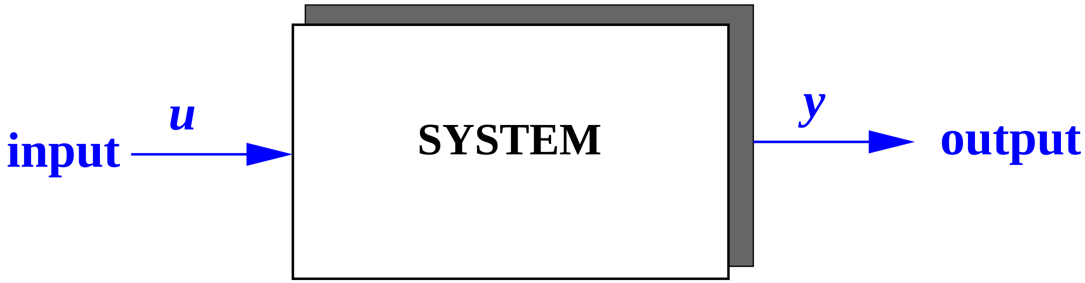
Mathematical description



Classical system theory and control:

u: input, y: output

Mathematical description



These models fail to deal with ‘initial conditions’.

A physical system is SELDOM an i/o map

Nevertheless... in numerous textbooks and ... Wikipedia

An input/output map?

Position history is not a 'function', a 'map', of force history.

Position depends not only on the force exerted, but also on initial conditions (initial position, velocity).

Around 1960: a paradigm shift

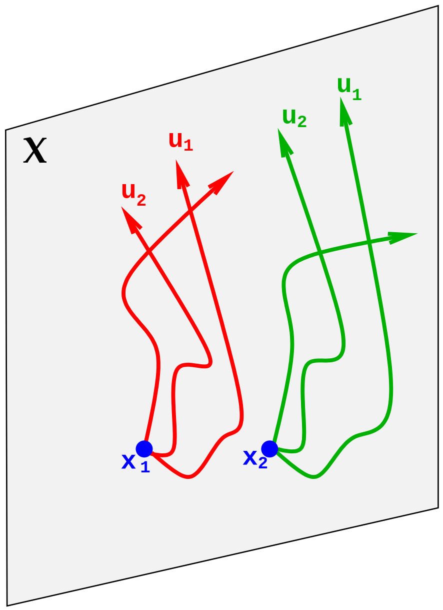
$$\rightsquigarrow \quad \frac{d}{dt}x = f(x,u), \; y = g(x,u)$$

Rudolf Kalman (1930-)

'Axiomatization'

State transition function:

$\phi(t, \mathrm{x}, u)$: **state reached at time t from x using input u.**



$$\frac{d}{dt}x = f(x, u), \; y = g(x, u)$$

Read-out function:

$g(\mathrm{x}, \mathrm{u})$: **output value with state x and input value u.**

‘Axiomatization’

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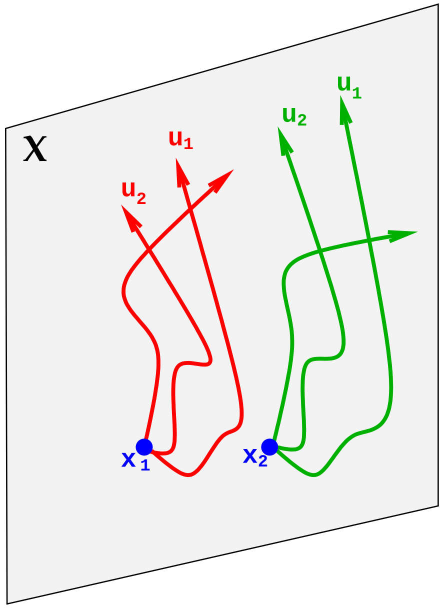


Simulation scenario:

set initial conditions, choose input function, run model.

Read-out function:

$g(\mathrm{x}, \mathrm{u})$: **output value with state x and input value u.**

Input/state/output systems

The input/state/output view turned out to be very effective and fruitful

- **for modeling dynamical systems (open)**
- **for control (stabilization, robustness, ...)**

Input/state/output systems

The input/state/output view turned out to be very effective and fruitful

- **for modeling dynamical systems (open)**
- **for control (stabilization, robustness, ...)**
- **signal processing, prediction, filtering**
- **learning, adaptation**
- **system identification: dynamic models from data**
- **many technological applications, control engineering, signal processing, technology,...**
- **etc., etc., etc.**

Input/state/output systems

Around 1960: a paradigm shift

$$\rightsquigarrow \quad \frac{d}{dt}x = f(x,u), \; y = g(x,u)$$

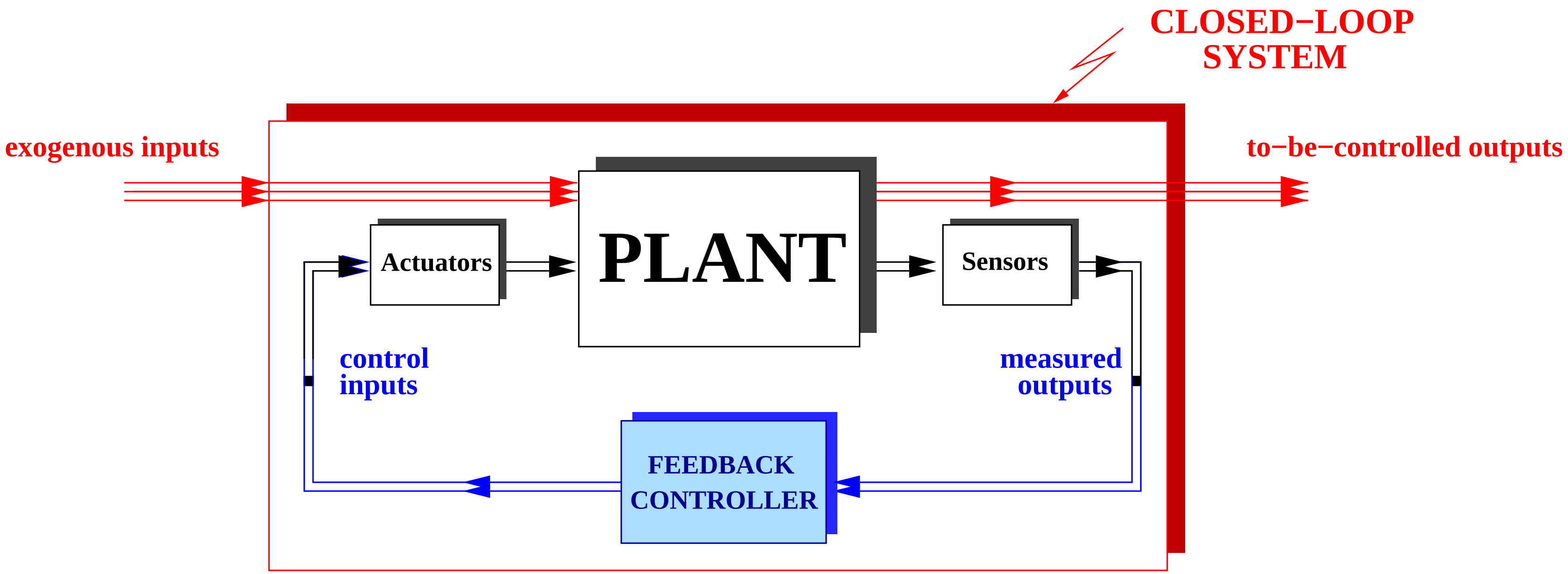
Excellent example of where a better mathematical framework stimulated engineering research.

Let’s take a closer look at the i/o framework ...

in control

Difficulties with i/o

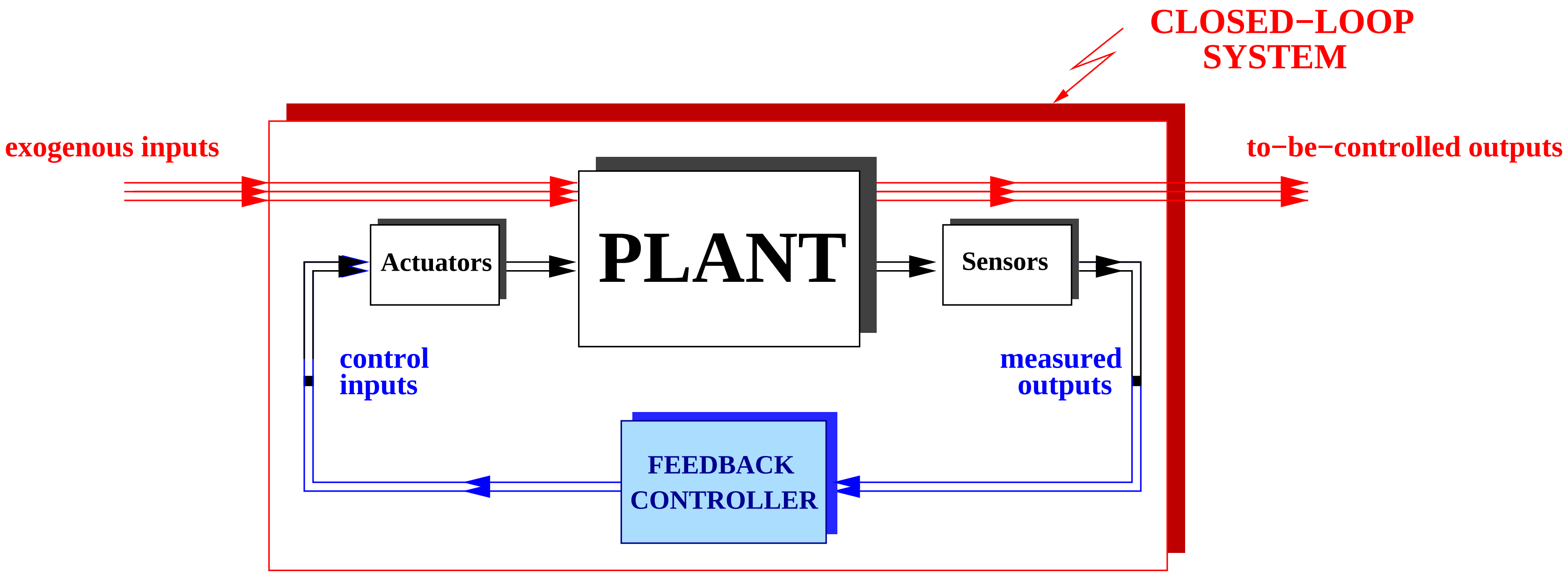
active control



Very intelligent, very useful, ... but general?

Difficulties with i/o

active control



versus passive control

Dampers, heat fins, pressure valves, grooves and strips, ...

Controllers without sensors and actuators

Difficulties with i/o

active control versus **passive** control

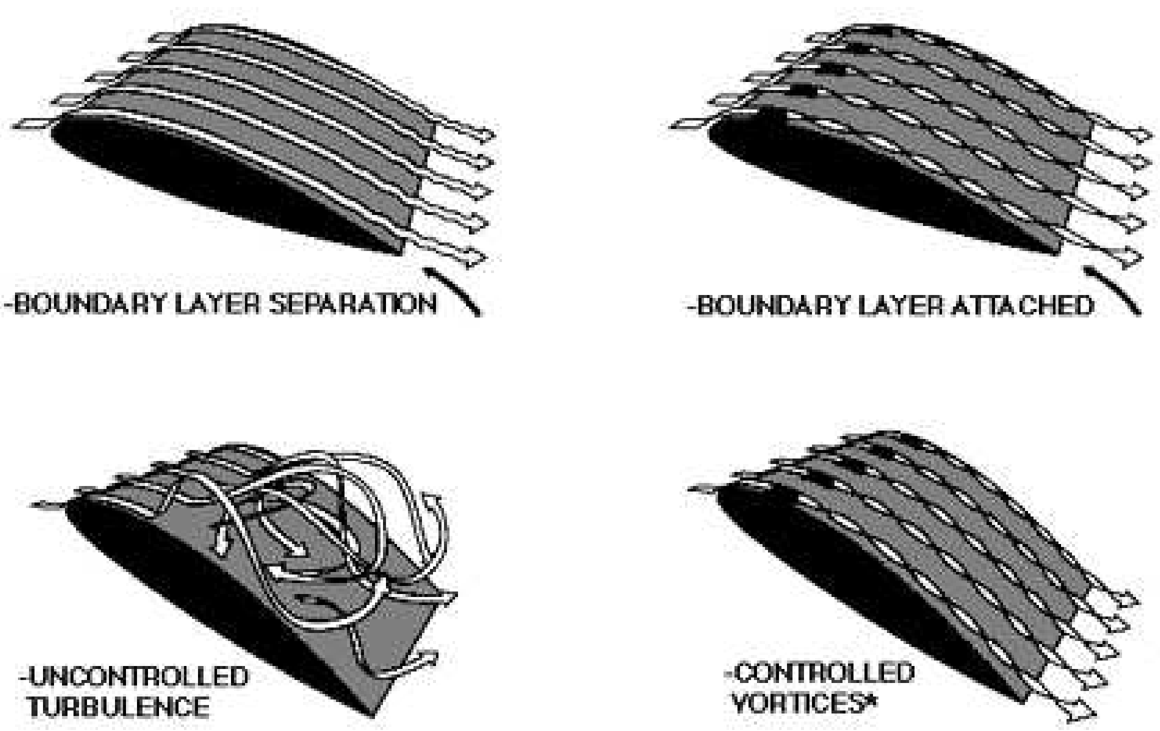
Controlling turbulence

airplanes, sharks, dolphins, golf balls, bicycling helmets, etc.

Difficulties with i/o

active control versus **passive** control

Controlling turbulence



Difficulties with i/o

active control versus passive control

Controlling turbulence

Nagano 1998

Difficulties with i/o

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Difficulties with i/o

active **control versus** **passive** **control**

Controlling turbulence

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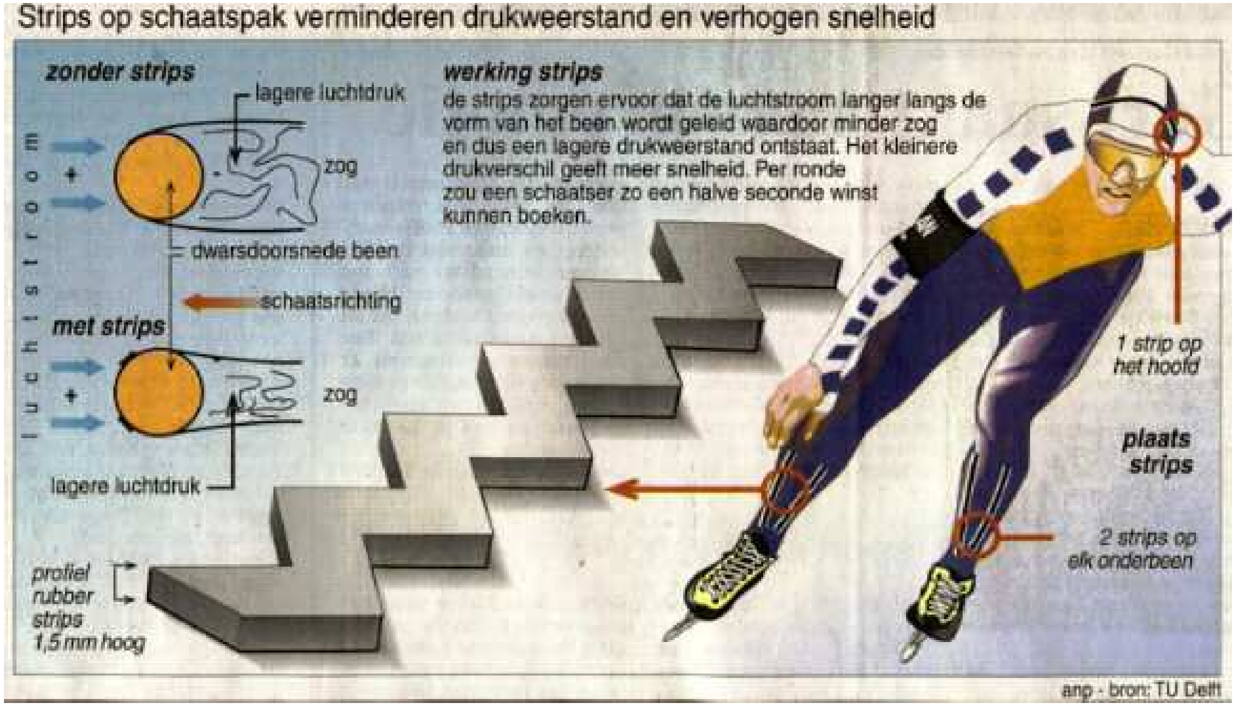


Difficulties with i/o

active **control versus** **passive** **control**

Controlling turbulence

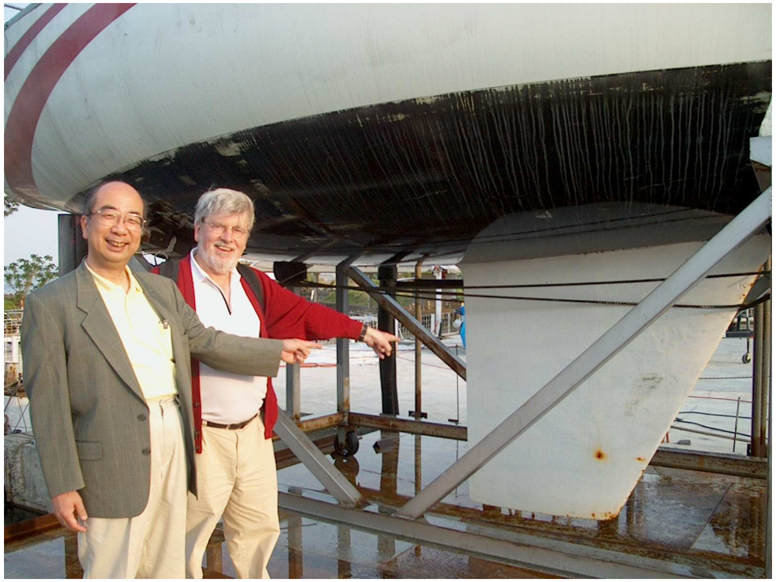
Nagano 1998

These are beautiful controllers!
But, in control, this is not viewed as “control” ...

Difficulties with i/o

active control versus passive control

Another example: the stabilizer of a ship



These are beautiful controllers!

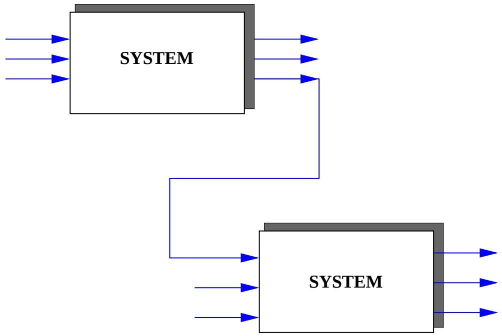
But, in control this is not called "stabilization" ...

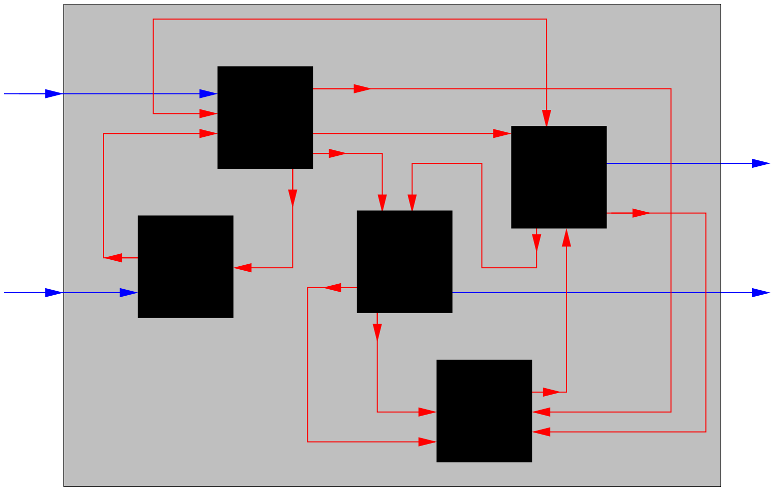
Let's take a closer look at the i/o framework ...

for interconnection

i/o and interconnection

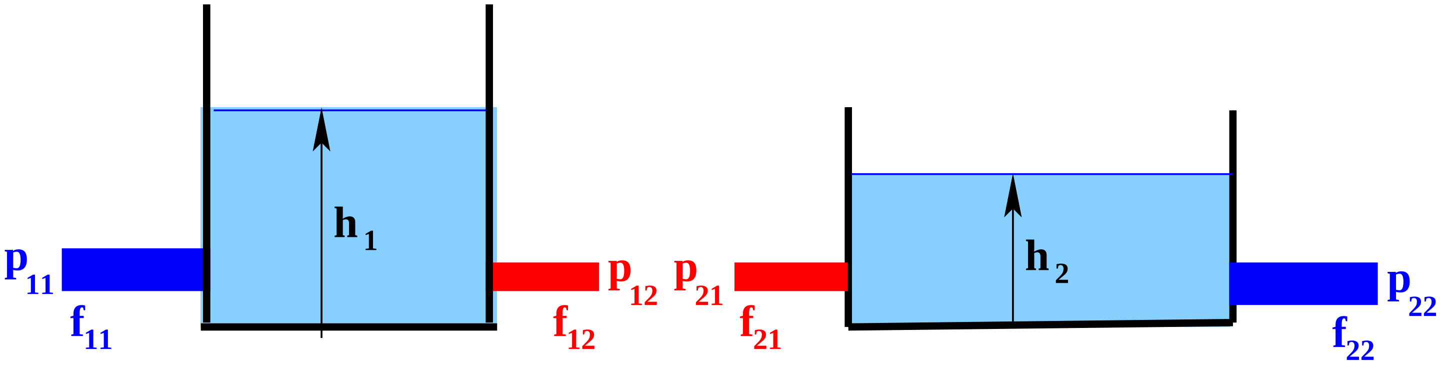
Interconnection:





Output-to-input assignment $\rightsquigarrow$ SIMULINK©

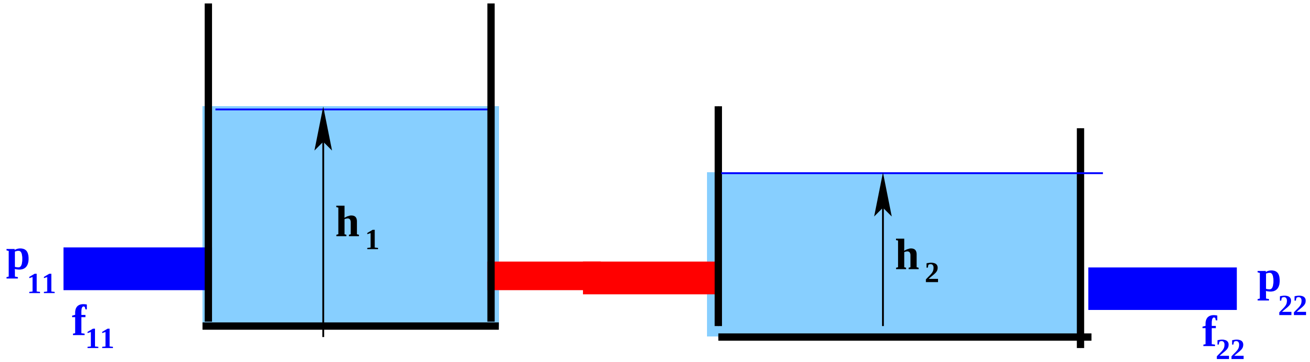
i/o and interconnection



inputs: the pressures $p_{11}, p_{12}, p_{21}, p_{22}$

outputs: the flows $f_{11}, f_{12}, f_{21}, f_{22}$

i/o and interconnection



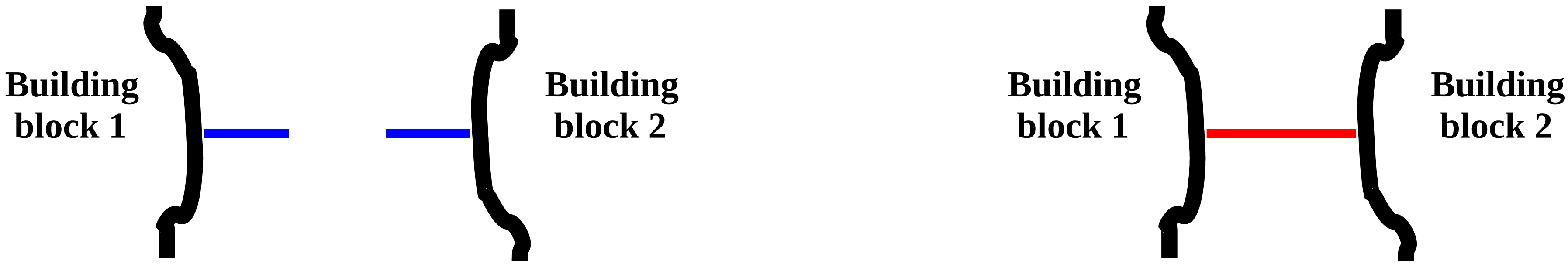
Interconnection: $p_{12} = p_{21}, f_{12} + f_{21} = 0$

Identifies 2 inputs AND (NOT WITH) 2 outputs,
the sort of thing SIMULINK© forbids.

This is the rule, not the exception (in hydraulics, mechanics, ...)
Interconnection is not input-to-output assignment!

Sharing variables, not input-to-output assignment,

is the mechanism by which physical systems interact.



Before interconnection:

variables on interconnected terminals are independent.

After interconnection: they are set equal.

No signal graphs!

Conclusion

The inability of the i/o framework to deal properly with

(i) interconnections

and

(ii) passive control

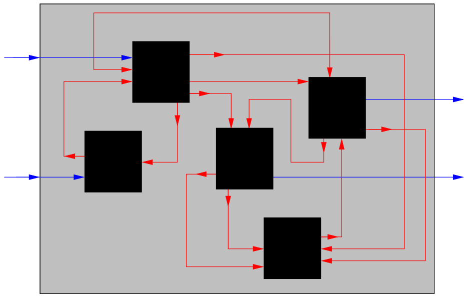
is a serious shortcoming. It just doesn't fit

Like the state, the input/output partition, if needed, should be constructed from first principles models. Contrary to the state, such a partition may not be useful, or even possible.

We need a better, more flexible, universal, framework that properly deals with

open & connected.

Conclusion



"Block diagrams unsuitable for serious physical modeling

- the control/physics barrier"

"Behavior based (declarative) modeling is a good alternative"

from K.J. Åström

Present Developments in Control Applications

IFAC 50-th Anniversary Celebration

Heidelberg, September 12, 2006.

General formalism

Generalities

What is a model? **As a mathematical concept.**

What is a dynamical system?

What is the role of differential equations in thinking about dynamical models?

Generalities

Intuition

We have a ‘phenomenon’ that produces ‘outcomes’ (‘events’).
We wish to model the outcomes that can occur.

Before we model the phenomenon:
the outcomes are in a set, which we call the *universum*.

After we model the phenomenon:
the outcomes are declared (thought, believed)
to belong to the *behavior* of the model,
a subset of this universum.

This subset =: a mathematical model.

Mathematization

This way we arrive at the

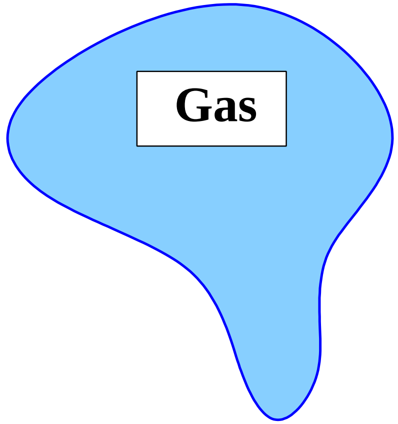
Definition

A *math. model* is a subset $\mathfrak{B}$ of a universum $\mathfrak{U}$ of outcomes

$$\mathfrak{B} \subseteq \mathfrak{U}.$$

$\mathfrak{B}$ is called the *behavior* of the model.

Mathematization



For example, the ideal gas law states that the temperature T, pressure P, volume V, and quantity (number of moles) N satisfy

$$\frac{PV}{NT} = R$$

with R a universal constant.

So, before Boyle, Charles, and Avogadro got into the act, T, P, V and N may have seemed unrelated, yielding

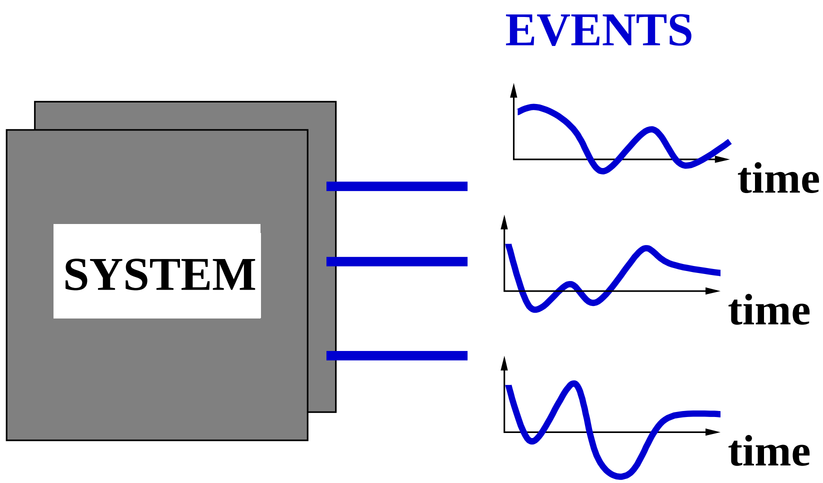
$$\mathfrak{U} = \mathbb{R}^4_+.$$

The ideal gas law restricts the possibilities to

$$\mathfrak{B} = \{(T, P, V, N) \in \mathbb{R}^4_+ \mid PV/NT = R\}$$

Dynamical systems

In dynamics, the outcomes are functions of time $\rightsquigarrow$



Which event trajectories are possible?

Dynamical systems

Definition

A dynamical system = $\Sigma := (\mathbb{T}, \mathbb{W}, \mathfrak{B})$

with $\mathbb{T} \subseteq \mathbb{R}$, the *time-axis* (= the relevant time instances),
$\mathbb{W}$, the *signal space*
(= where the variables take on their values),
$\mathfrak{B} \subseteq \mathbb{W}^{\mathbb{T}}$ *the behavior* (= the admissible trajectories).

Dynamical systems

Definition

A dynamical system = $\boxed{\Sigma := (\mathbb{T}, \mathbb{W}, \mathfrak{B})}$

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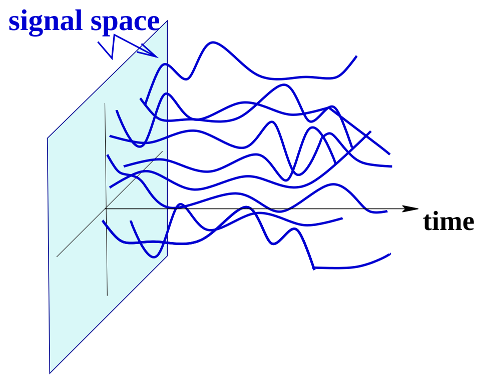


Totality of ‘legal’ trajectories =: the behavior

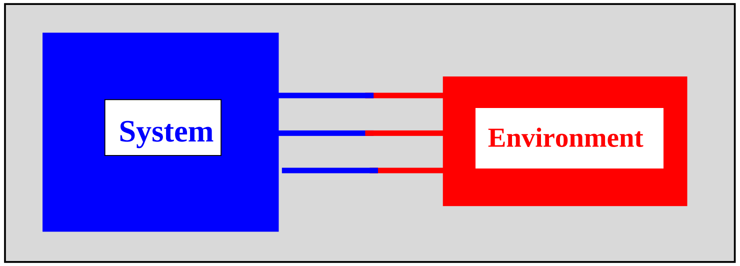
Dynamical Systems

The behavior is all there is

Equivalence, representations, symmetries, controllability, model simplification, etc. must refer to the behavior.

Simulation scenario: **choose model for environment**
initial conditions of system
initial conditions for environment

run the full model.



Dynamical Systems

The behavior is all there is

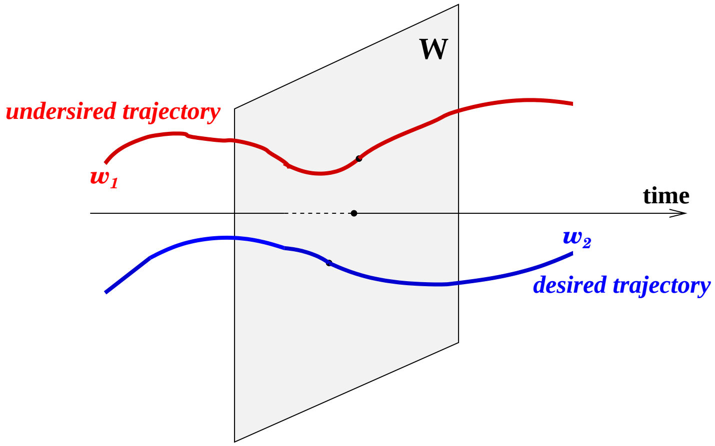
Equivalence, representations, symmetries, controllability, model simplification, etc. must refer to the behavior.

In the mean time there is a rich theory of dynamical systems with behaviors as central notion.
But, compared to flows and input/state/output systems, this theory is *in statu nascendi*.

Controllability

Controllability

Take any two trajectories $w_1, w_2 \in \mathfrak{B}$.



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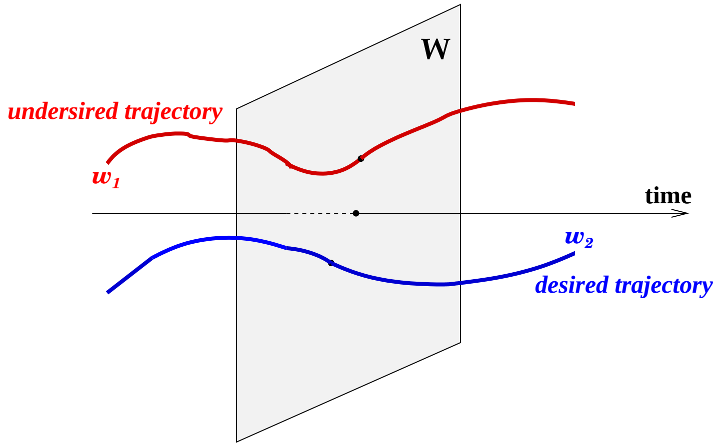
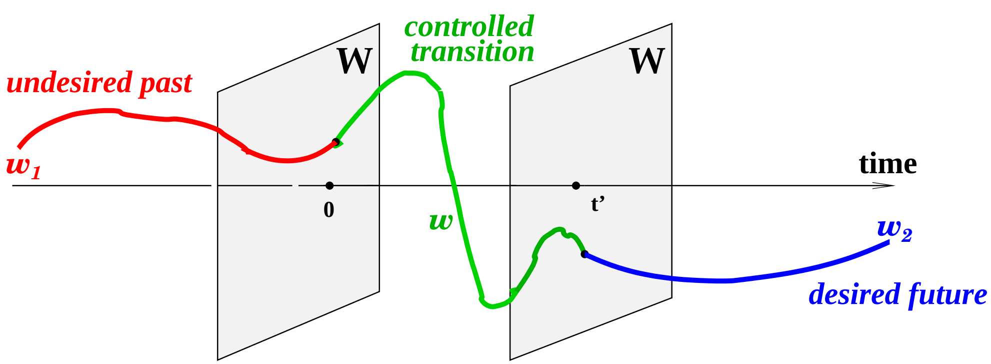


'Controllability':



PDE’s

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Much of the theory also applies to PDE's.

$\mathbb{T} = \mathbb{R}^{\mathtt{n}}$, **the set of independent variables, often** $\mathtt{n} = 4$,
$\mathbb{W} = \mathbb{R}^{\mathtt{w}}$, **(say) the set of dependent variables,**
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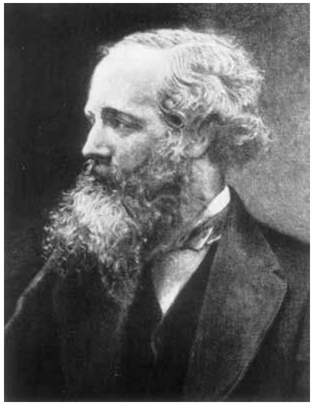
So, the behavior consists of a family of functions of time and space, i.e. 'fields'.

Elasticity, continuum mechanics
Waves
Heat diffusion
etc.

Example

Maxwell's eq'ns, diffusion eq'n, wave eq'n, . . .



$$\begin{aligned} \nabla \cdot \vec{E} &= \frac{1}{\varepsilon_0}\rho\,, \\ \nabla \times \vec{E} &= -\frac{\partial}{\partial t}\vec{B}, \\ \nabla \cdot \vec{B} &= 0\,, \\ c^2\nabla \times \vec{B} &= \frac{1}{\varepsilon_0}\vec{j} + \frac{\partial}{\partial t}\vec{E}. \end{aligned}$$

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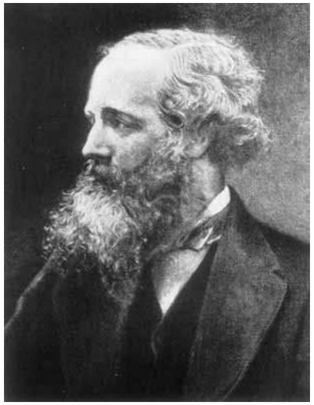

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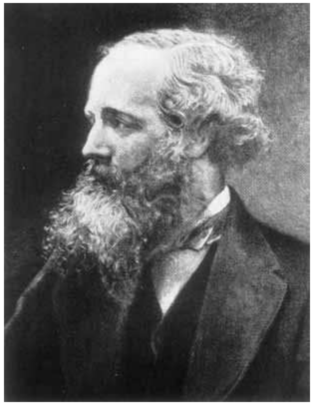

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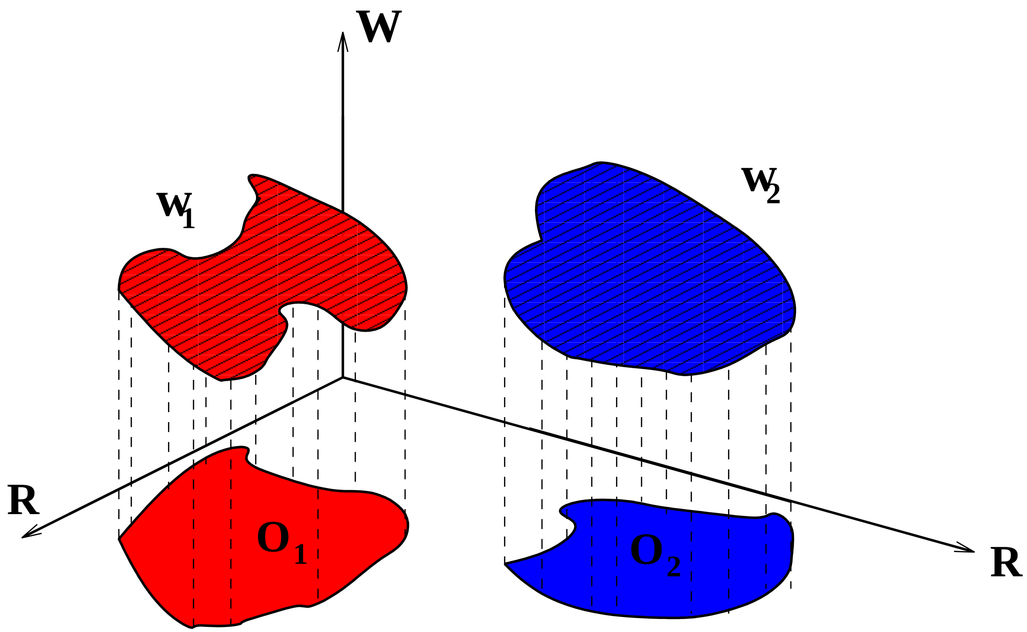
PDE's, as Maxwell's equations, define open systems,
but not input/output,
& the notion of state has not been thought out.

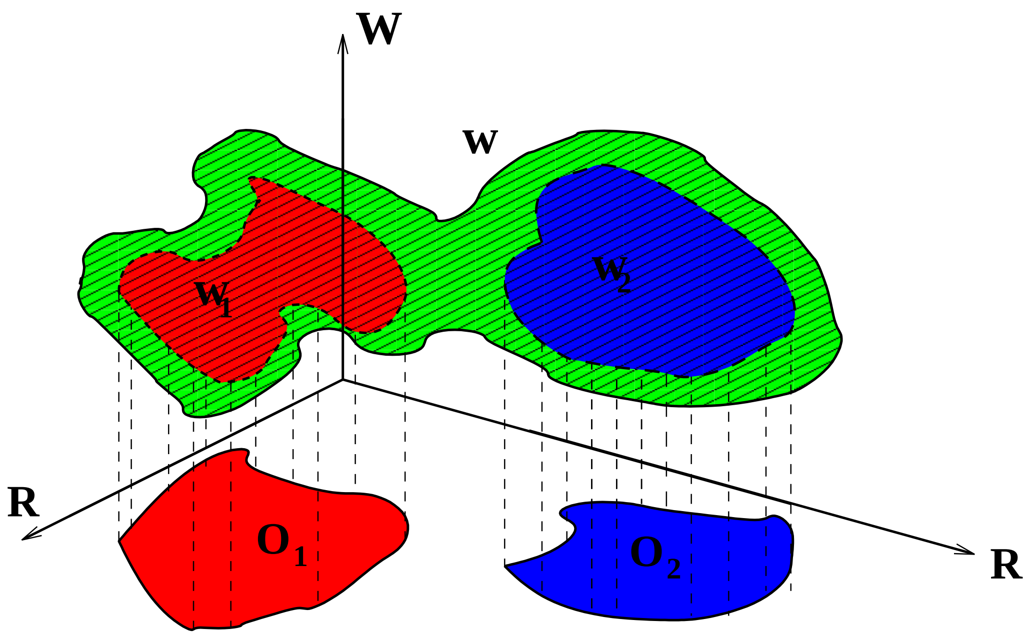
Controllability def'n in pictures:



$$w_1, w_2 \in \mathfrak{B}.$$

Controllability for PDE's

$w \in \mathfrak{B}$ 'patches' $w_1, w_2 \in \mathfrak{B}$.



Controllability :⇔ 'patch-ability'.

Are Maxwell’s equations controllable ?

Are Maxwell's equations controllable ?

YES, they are indeed!

controllability $\Leftrightarrow$ there exists a potential!

Summary

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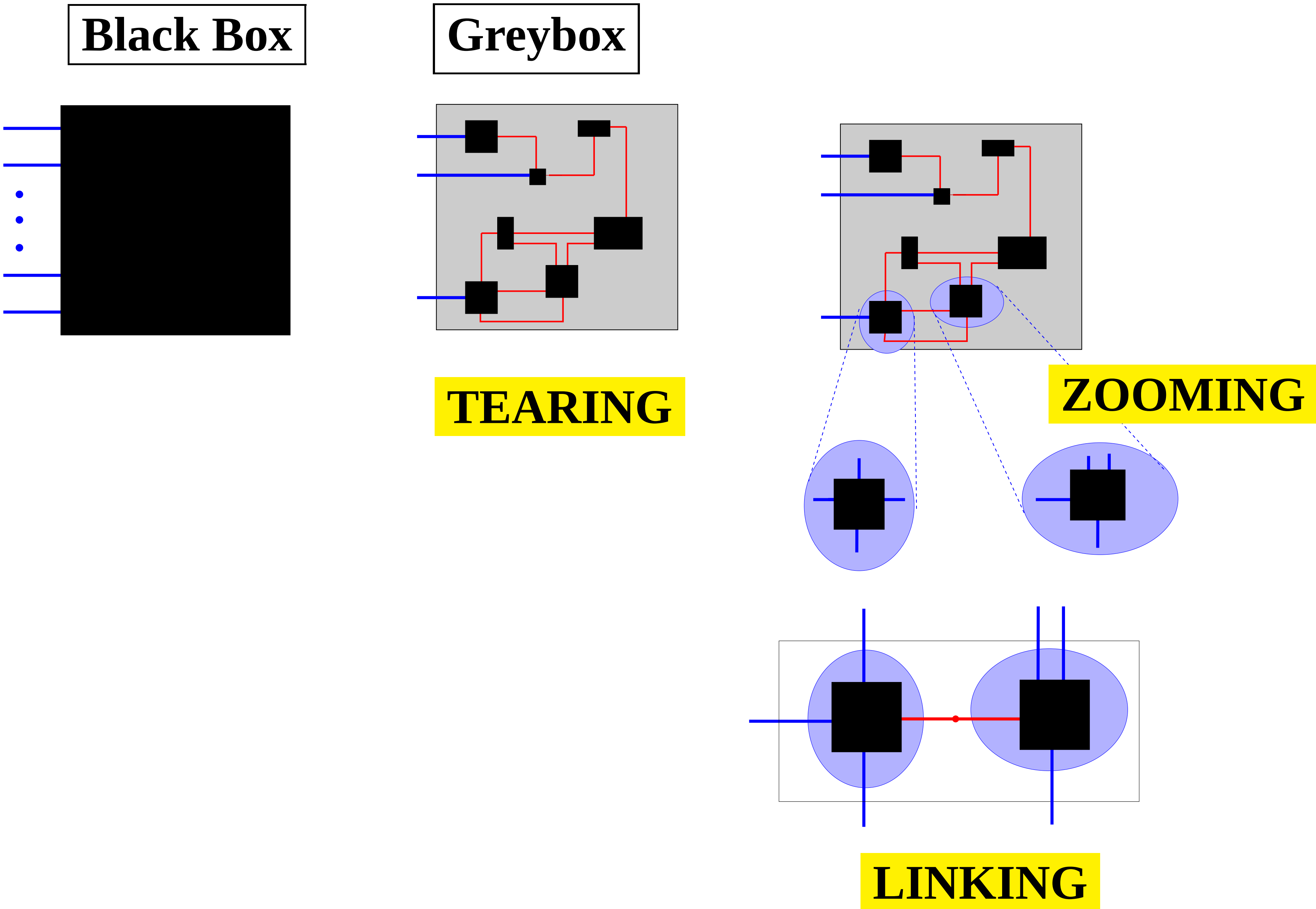
- **Behavioral modeling of open systems, with interconnection as variable sharing, tearing-zooming-linking,**

gets the physics right

Summary



Details & copies of the lecture frames are available from/at

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Thank you

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