PORTS and TERMINALS

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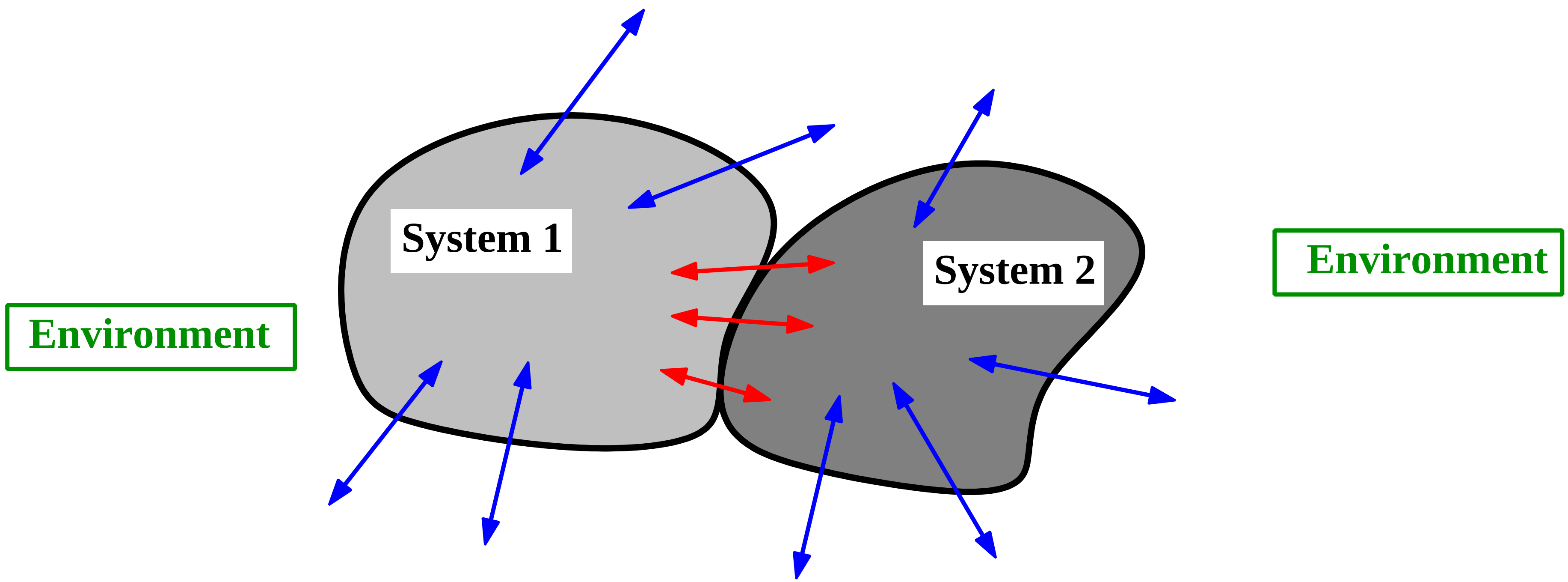
Theme: energy transfer



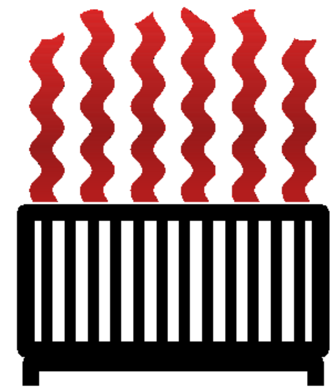
How is energy transferred from the environment to a system?

How is energy transferred between systems?

Does interconnection mean energy transfer?

Energy

Energy **:= a physical quantity transformable into heat.**



Energy

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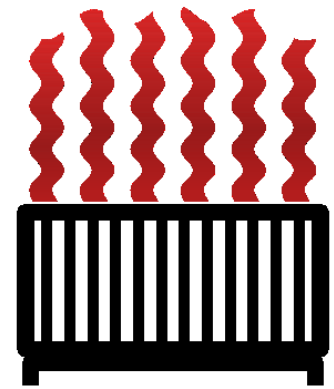
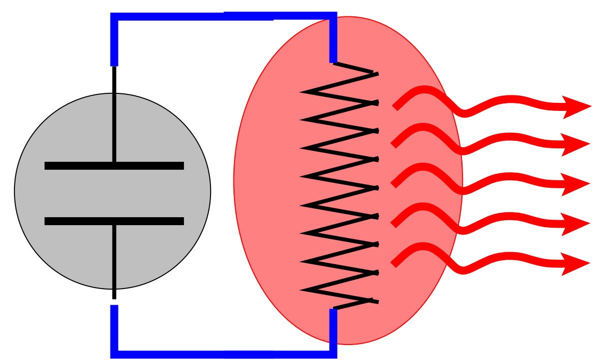


For example, capacitor $\mapsto$ resistor $\mapsto$ heat.

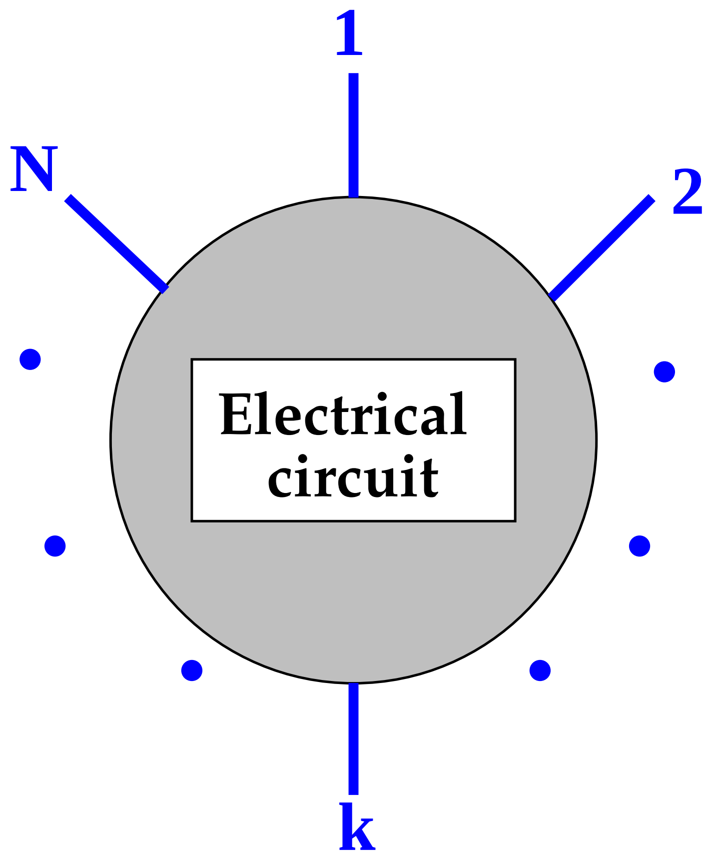
Energy on capacitor = $\frac{1}{2}CV^2$



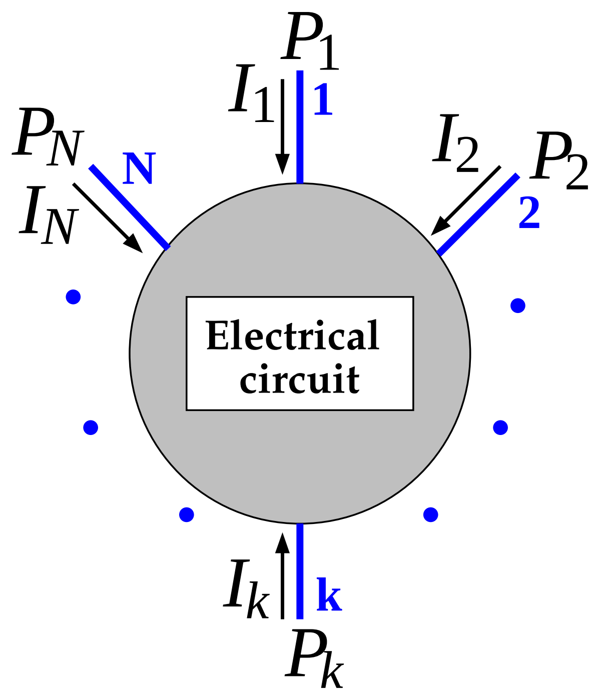
Electrical ports

Electrical circuit

wires $\cong$ terminals



Electrical circuit



At each terminal:

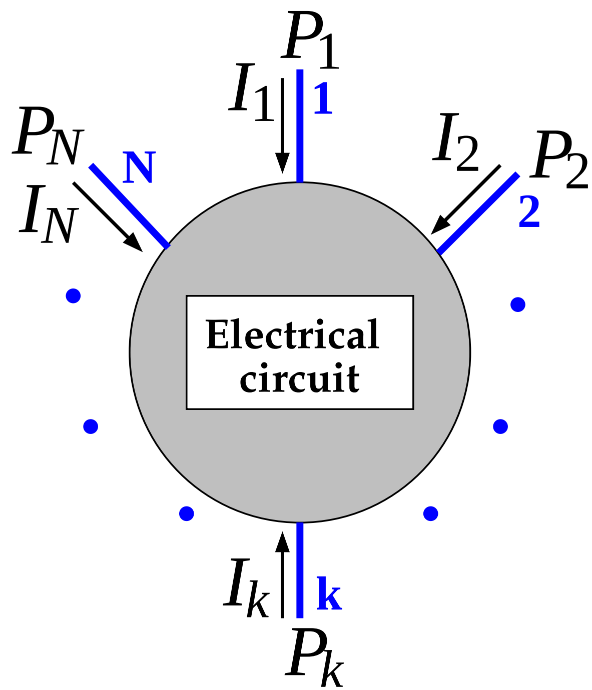
a current (counted > 0 into the circuit) and a potential

$$\rightsquigarrow \textbf{behavior } \mathscr{B} \subseteq \left(\mathbb{R}^N \times \mathbb{R}^N\right)^{\mathbb{R}}$$

$(I_1, I_2, \ldots, I_N, P_1, P_2, \ldots, P_N) : \mathbb{R} \to \mathbb{R}^N \times \mathbb{R}^N \in \mathscr{B}$ **means:**

this current/potential trajectory is compatible with the circuit architecture and its element values.

KCL and KVL



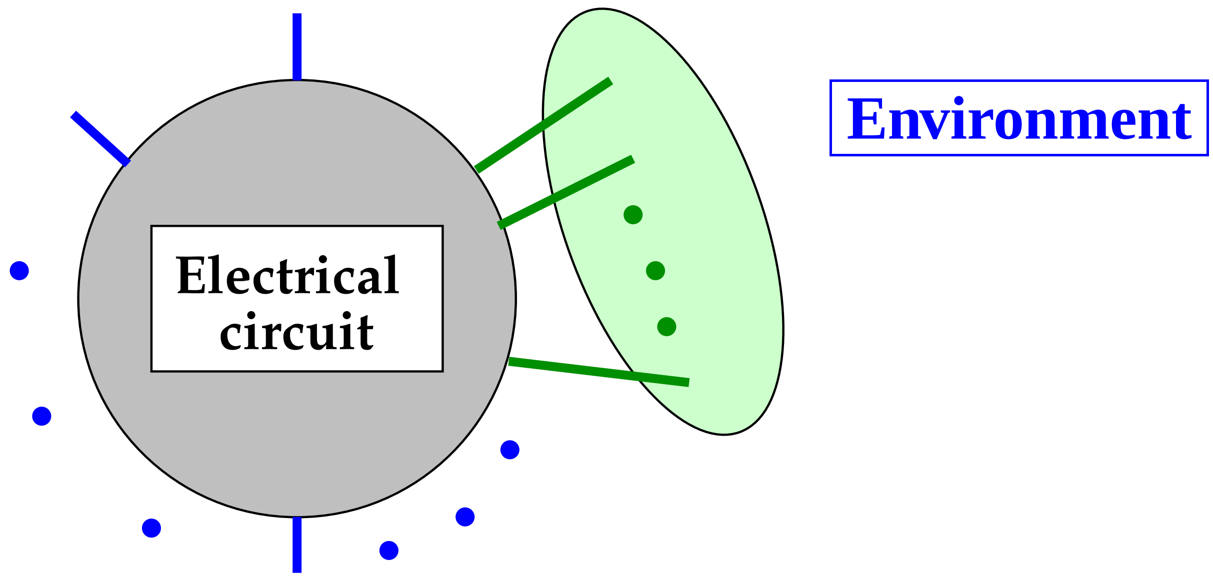
Kirchhoff's current law (KCL):

$$[\![(I_1, I_2, \ldots, I_N, P_1, P_2, \ldots, P_N) \in \mathscr{B}]\!] \Rightarrow [\![\ I_1 + I_2 + \cdots + I_N = 0\ ]\!].$$

Kirchhoff's voltage law (KVL):

$$[\![(I_1, I_2, \ldots, I_N, P_1, P_2, \ldots, P_N) \in \mathscr{B} \textbf{ and } \alpha : \mathbb{R} \to \mathbb{R}\ ]\!]$$
$$\Rightarrow [\![\ (I_1, I_2, \ldots, I_N, P_1 + \alpha, P_2 + \alpha, \ldots, P_N + \alpha) \in \mathscr{B}\ ]\!].$$

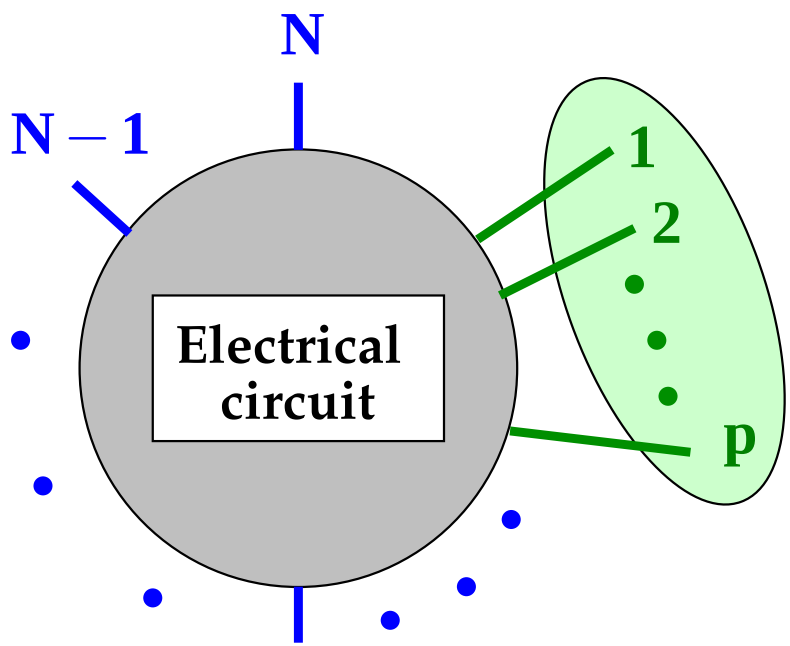
Energy transfer



Assume that we monitor the current/potential on a set of terminals.

Can we speak about *'the energy transferred from the environment to the circuit along these terminals'*?

Ports



Terminals $\{1, 2, \ldots, p\}$ **form a port** $:\Leftrightarrow$

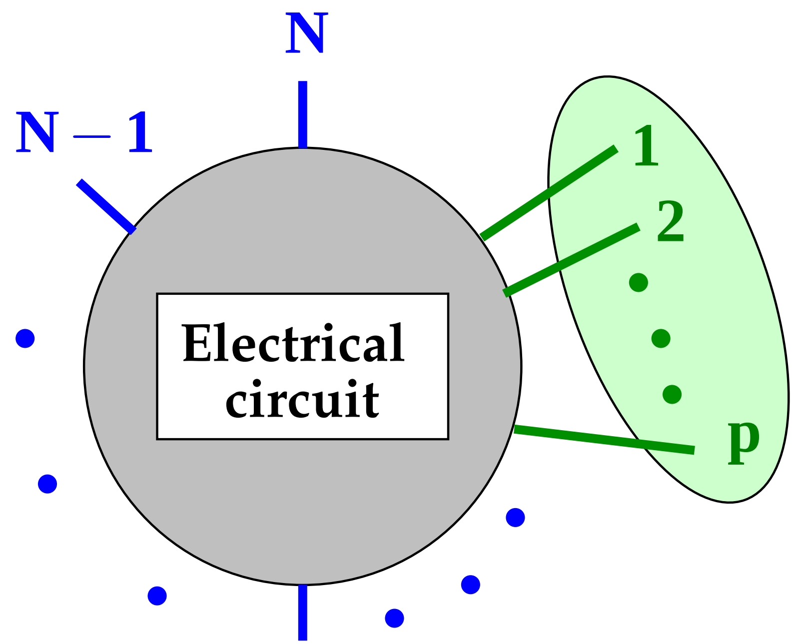
$$[\![(I_1, \ldots, I_p, I_{p+1}, \ldots, I_N, P_1, \ldots, P_p, P_{p+1}, \ldots, P_N,) \in \mathscr{B}]\!]$$

$$\Rightarrow \quad [\![I_1 + I_2 + \cdots + I_p = 0]\!].$$

'port KCL'

KCL $\Rightarrow$ all terminals together form a port.

Ports



If terminals $\{1, 2, \ldots, p\}$ **form a port, then**

power in = $P_1(t)I_1(t) + P_2(t)I_2(t) + \cdots + P_p(t)I_p(t)$

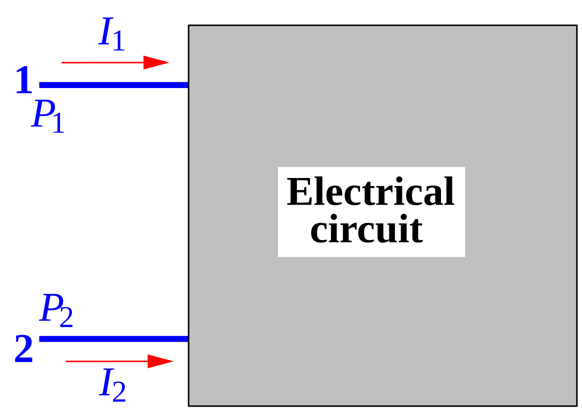
energy in = $\int_{t_1}^{t_2} [P_1(t)I_1(t) + P_2(t)I_2(t) + \cdots + P_p(t)I_p(t)]\, dt$

This interpretation in terms of power and energy is not valid unless these terminals form a port !

Examples

2-terminal 1-port devices:

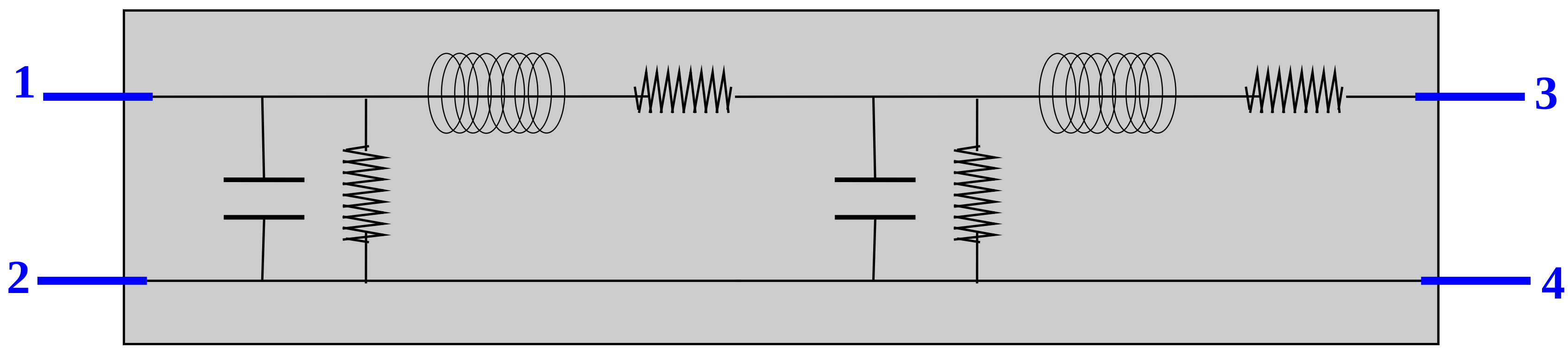
resistors, inductors, capacitors, memristors, etc., any 2-terminal circuit composed of these.



KCL $\Rightarrow$ **a port** $(I_1 = -I_2 =: I)$.
KVL $\Rightarrow$ **only** $P_1 - P_2 =: V$ **matters.**

$\rightsquigarrow$ **usual circuit variables** (I, V)**.**

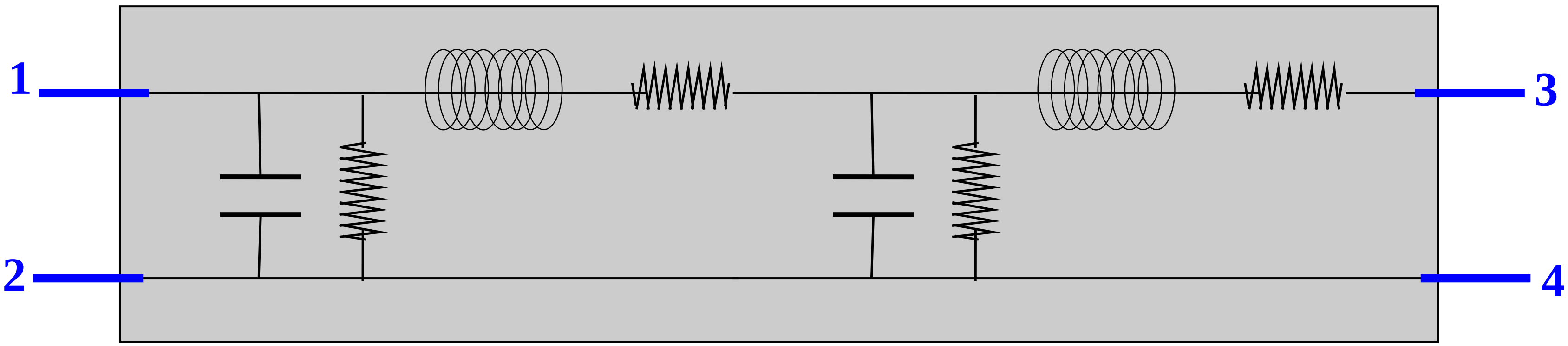
Example



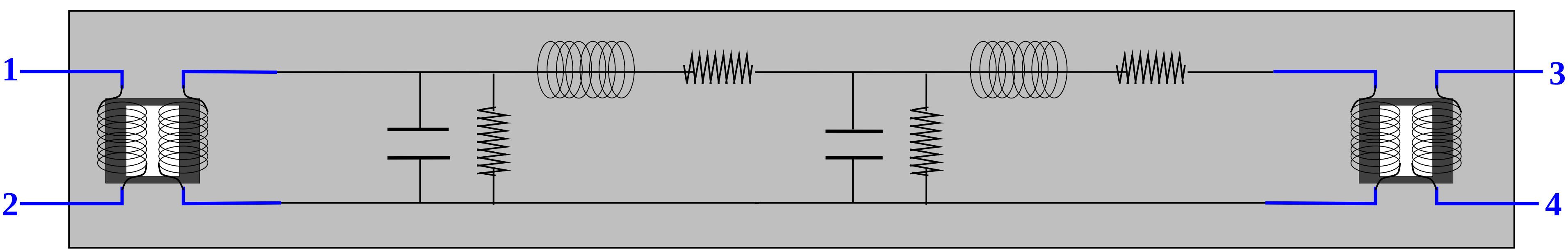
Terminals $\{1, 2, 3, 4\}$ form a port. But $\{1, 2\}$ and $\{3, 4\}$ do not.

We cannot speak about
***'the energy transferred from terminals $\{1, 2\}$ to $\{3, 4\}$'*.**

Example

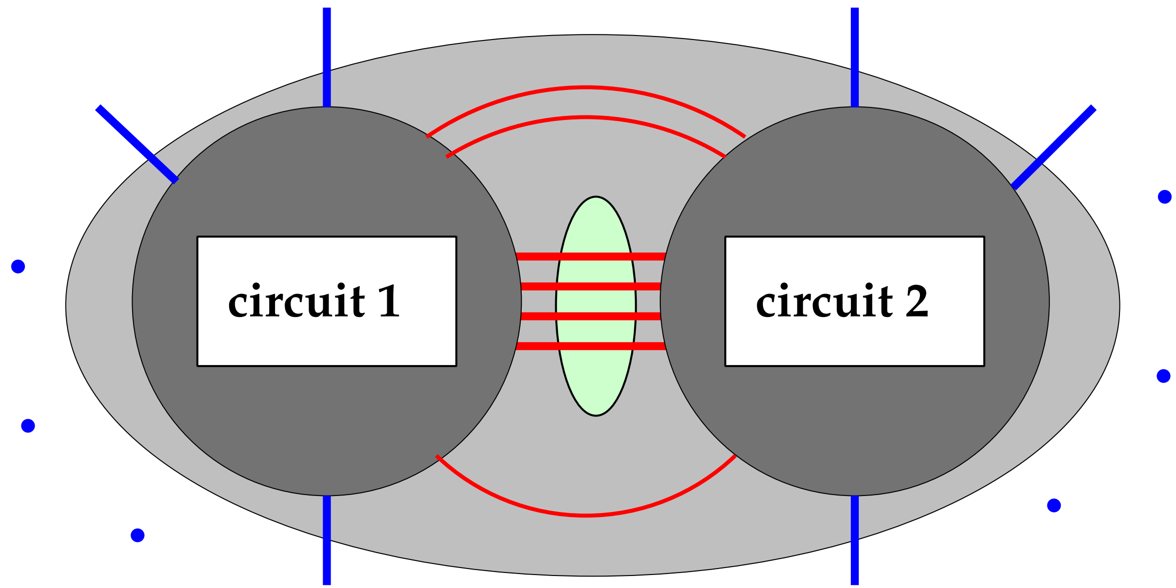


Terminals $\{1,2,3,4\}$ form a port. But $\{1,2\}$ and $\{3,4\}$ do not.



Terminals $\{1,2\}$ and $\{3,4\}$ form ports.

Energy transfer between circuits

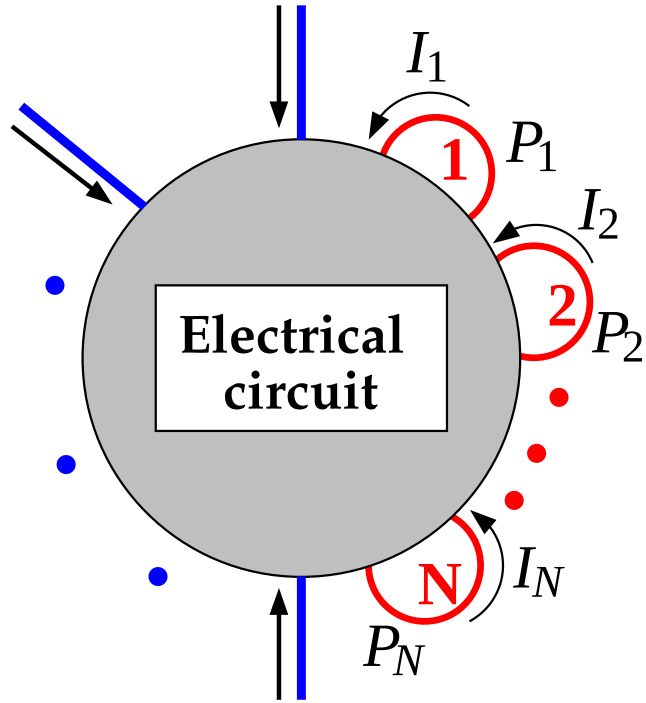


Assume that we monitor the current/potential on a set of terminals between circuits or within a circuit.

Can we speak about

***'the energy transferred along these terminals'*?**

Internal ports



Terminals $\{1,2,\ldots,N\}$ **form an internal port** $:\Leftrightarrow$

$$[\![(I_1, I_2, \ldots, I_N, P_1, P_2, \ldots, P_N) \in \mathscr{B}]\!]$$
$$\Rightarrow [\![I_1 + I_2 + \cdots + I_N = 0]\!].$$

'internal port-KCL'

Power and energy

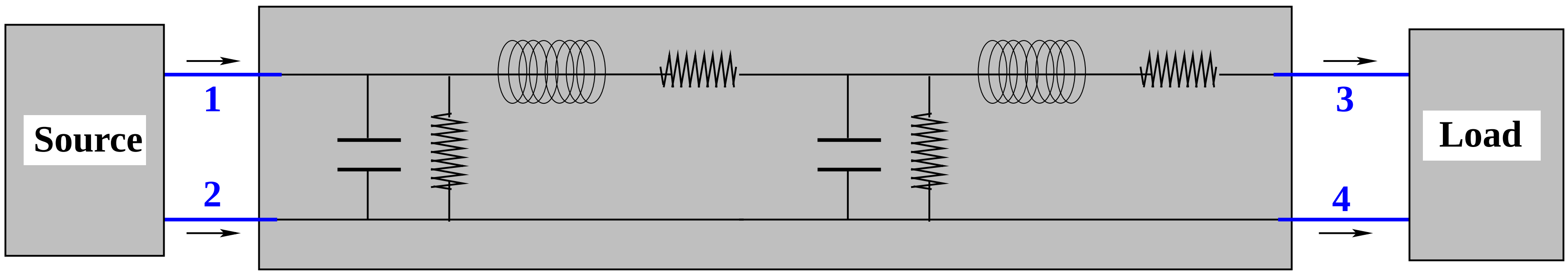
Flow through the terminals ***from one side to the other*** **in the direction of the arrows:**

power = $I_1(t)P_1(t) + I_2(t)P_2(t) + \cdots + I_N(t)P_N(t)$

energy = $\int_{t_1}^{t_2} [I_1(t)P_1(t) + I_2(t)P_2(t) + \cdots + I_N(t)P_N(t)]\, dt$

This physical interpretation of power and energy is valid only if the terminals form an internal port.

Example



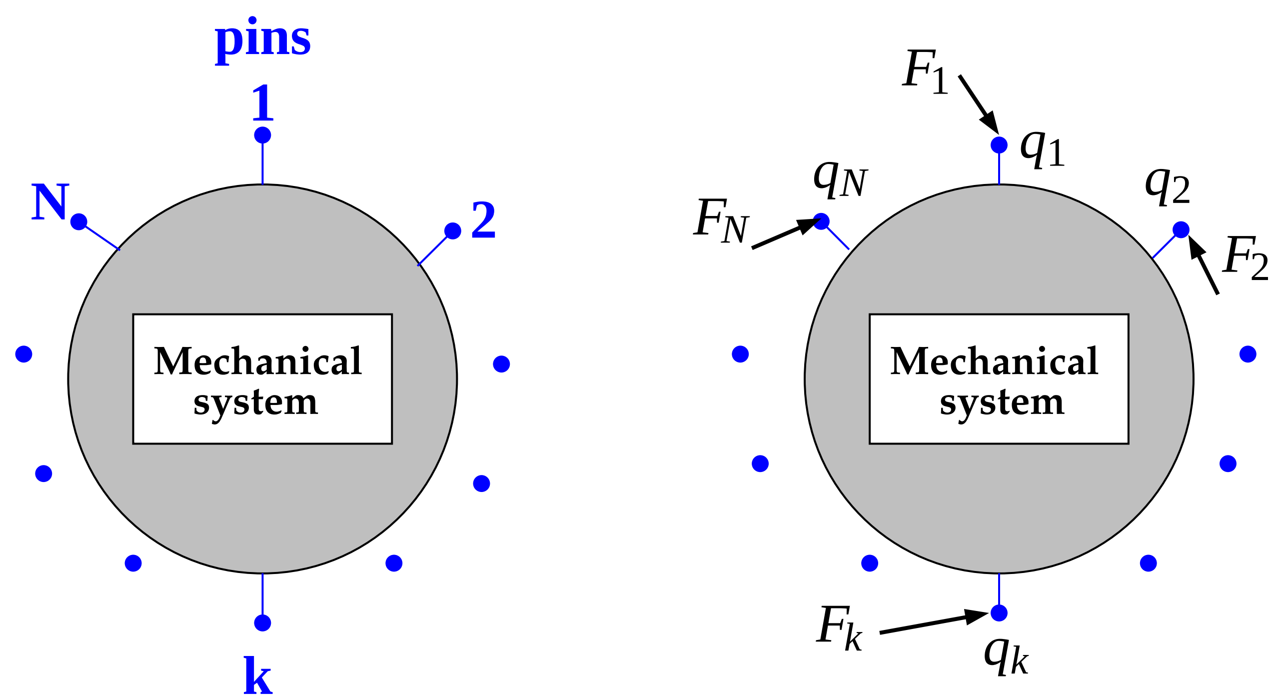
Because of the source and the load (2-terminal 1-ports) terminals $\{1, 2\}$ and $\{3, 4\}$ form internal ports.

Therefore, we can speak of

***'the energy transferred from the source to the load'*.**

Mechanical ports

Mechanical systems

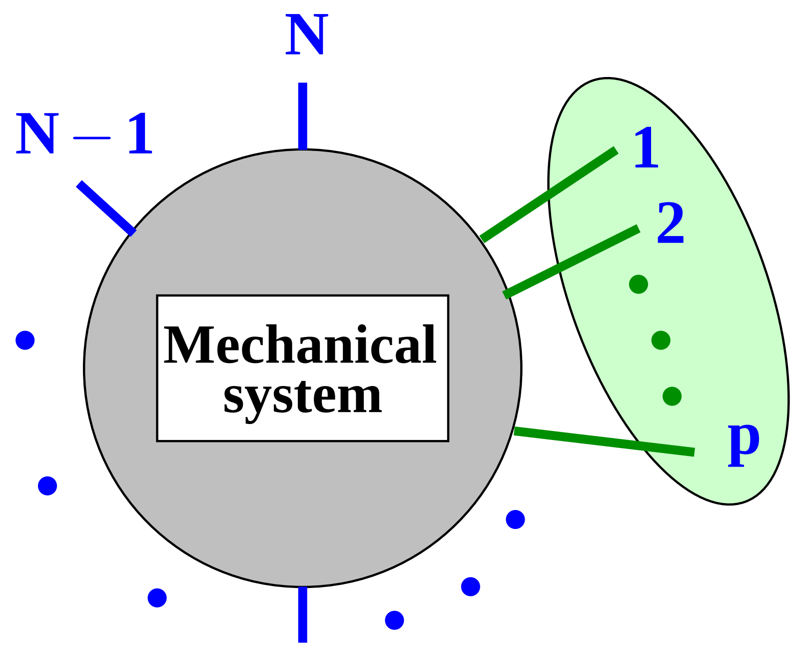


At each terminal: a position and a force.

$\rightsquigarrow$ **position/force trajectories** $(q, F) \in \mathscr{B} \subseteq ((\mathbb{R}^\bullet)^{2N})^{\mathbb{R}}$.

More generally, a position, force, angle, and torque.

Mechanical ports



Terminals $\{1, 2, \ldots, p\}$ **form a (mechanical) port** $:\Leftrightarrow$

$$(q_1, \ldots, q_p, q_{p+1}, \ldots, q_N, F_1, \ldots, F_p, F_{p+1}, \ldots, F_N) \in \mathscr{B},$$

$$\Rightarrow \quad [\![F_1 + F_2 + \cdots + F_p = 0]\!].$$

'port KFL'

Power and energy

If terminals $\{1, 2, \ldots, p\}$ form a port, then

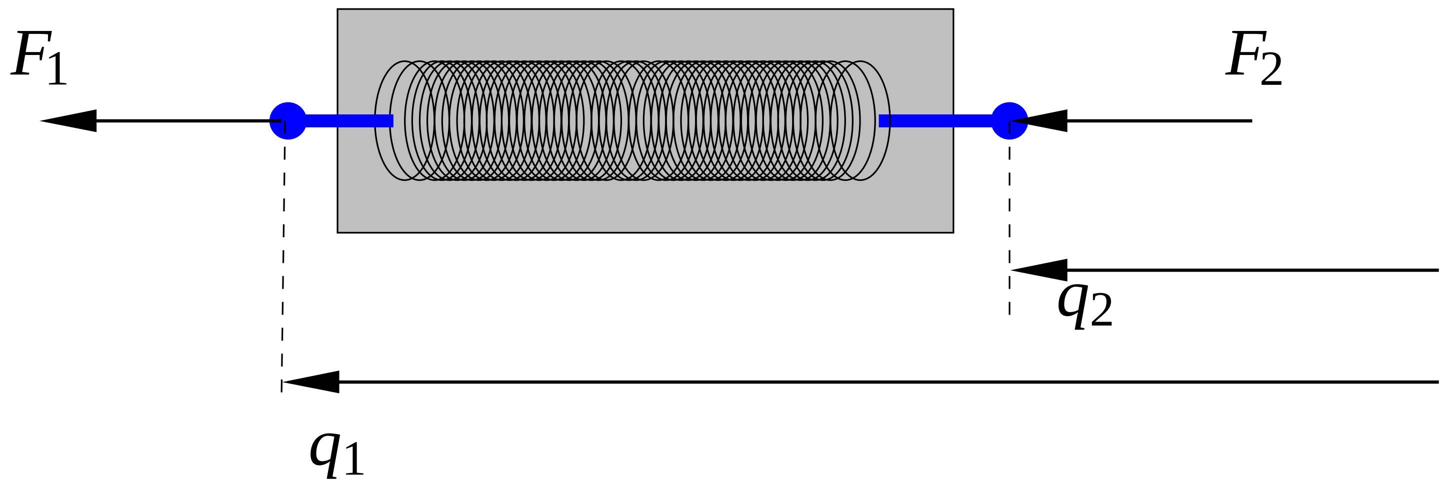
$$\textbf{power in} \;=\; F_1(t)^\top \tfrac{d}{dt} q_1(t) + \cdots + F_p(t)^\top \tfrac{d}{dt} q_p(t),$$

$$\textbf{energy in} \;=\; \int_{t_1}^{t_2} \left(F_1(t)^\top \frac{d}{dt} q_1(t) + \cdots + F_p(t)^\top \frac{d}{dt} q_p(t) \right) dt.$$

This interpretation in terms of power and energy is not valid
unless these terminals form a port !

Example

Spring



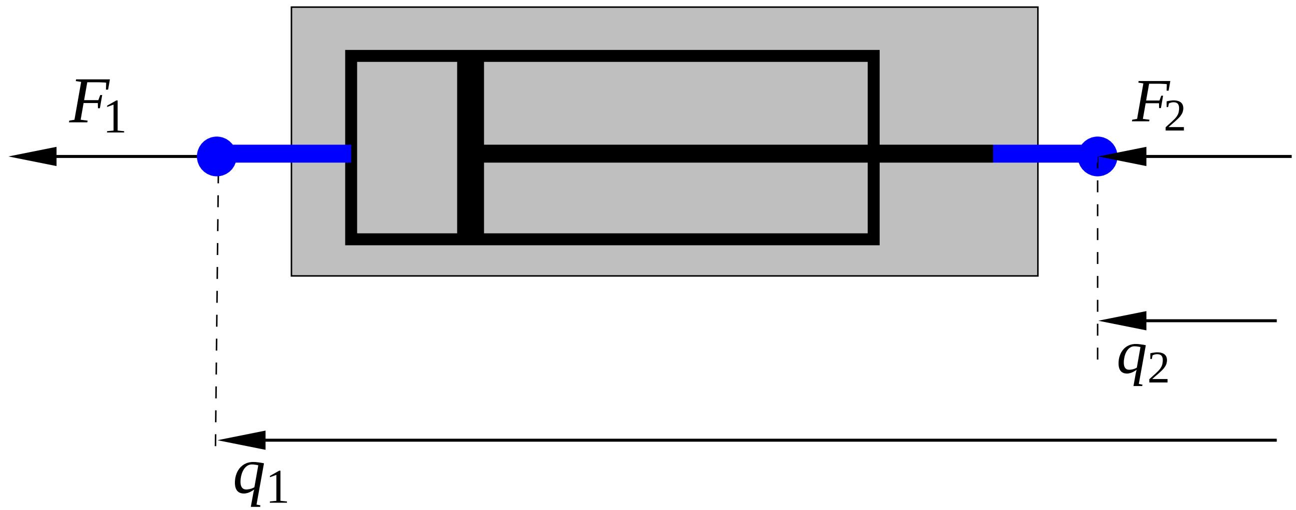
$$F_1 + F_2 = 0, \quad K(q_1 - q_2) = F_1$$

satisfies KFL

power in $= F_1(t)\frac{d}{dt}q_1(t) + F_2(t)\frac{d}{dt}q_2(t) = F_1(t)\frac{d}{dt}(q_1 - q_2)(t)$

Examples

Damper

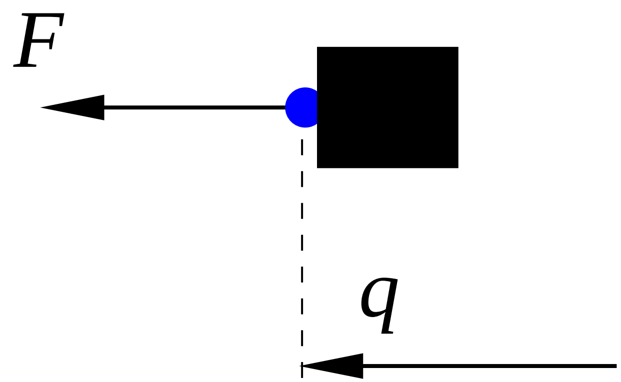


$$F_1 + F_2 = 0, \quad D\frac{d}{dt}(q_1 - q_2) = F_1$$

satisfies KFL

Springs and dampers, and their interconnection form ports.

A mass



$$M\frac{d^2}{dt^2}q = F$$

does **not** satisfy KFL

Not a port!!!

Therefore $F(t)\frac{d}{dt}q(t)$ **is not power**
(even though it has the dimension of power).

Consequences

Consequences of the fact that a mass is not a port.

- **The inerter:**

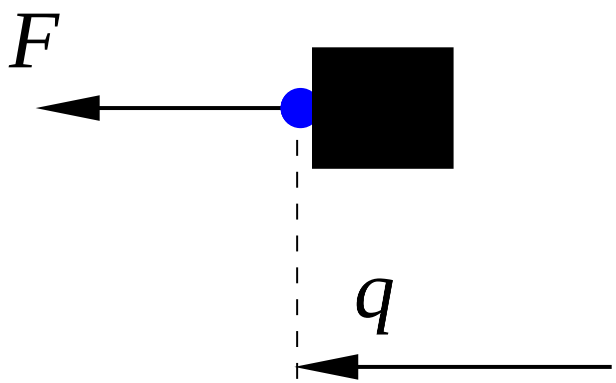
  RLC synthesis $\Leftrightarrow$ Damper-Spring-Inerter synthesis

  $\not\Leftrightarrow$ Damper-Spring-Mass synthesis

- **Motion energy**
- **Energy as an extensive quantity**

Motion energy

Back to the mass

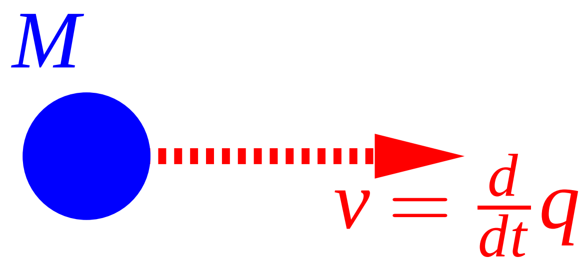


$$M\frac{d^2}{dt^2}q = F \quad \Rightarrow \quad \frac{d}{dt}\frac{1}{2}M||\frac{d}{dt}q||^2 = F^\top\frac{d}{dt}q$$

If $F^\top\frac{d}{dt}q$ ***is not power,***

is $\frac{1}{2}M||\frac{d}{dt}q||^2$ ***not stored (kinetic, motion) energy ???***

Kinetic energy and invariance under uniform motions



What is the kinetic energy?

$$\mathscr{E}_{\textbf{kinetic}} = \frac{1}{2} M \, ||v||^2$$

Willem ’s Gravesande
1688–1742

Émilie du Châtelet
1706–1749

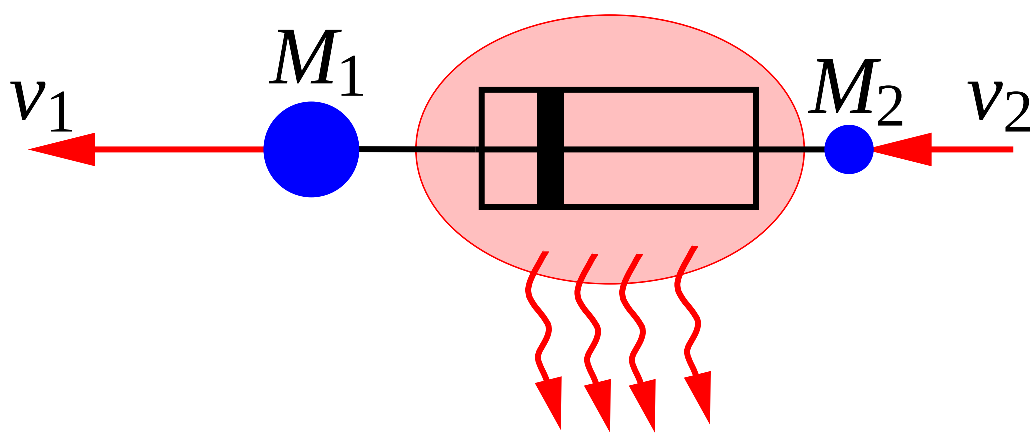
This expression is not invariant under uniform motion.
Physical significance dubious!

Motion energy

What is the motion energy?

What quantity is transformable into heat?

Calculate by considering



Motion energy

What is the motion energy?

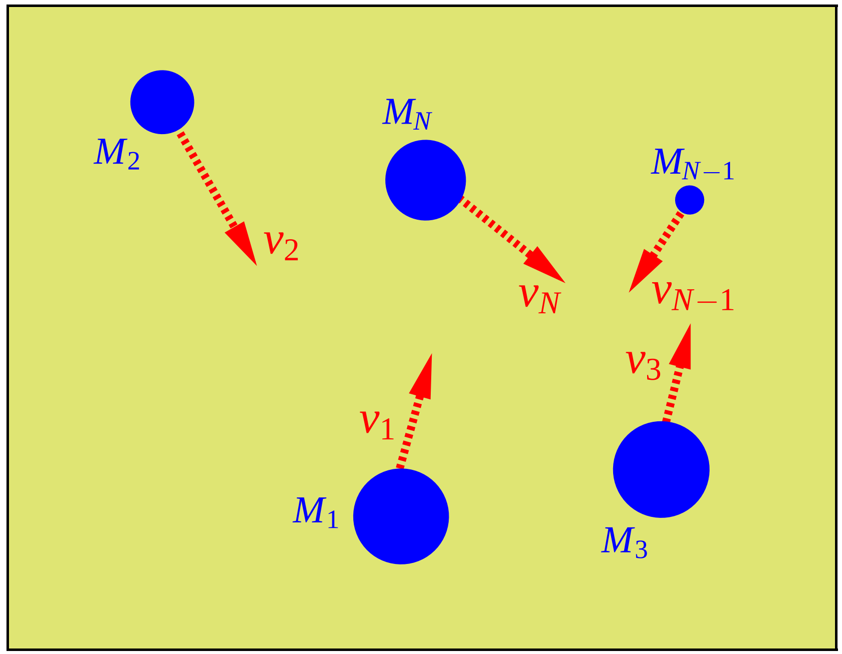
What quantity is transformable into heat?

$$\mathcal{E}_{\textbf{motion}} = \frac{1}{2}\,\frac{M_1\,M_2}{M_1+M_2}\,||v_1 - v_2||^2$$

Invariant under uniform motion.

Motion energy

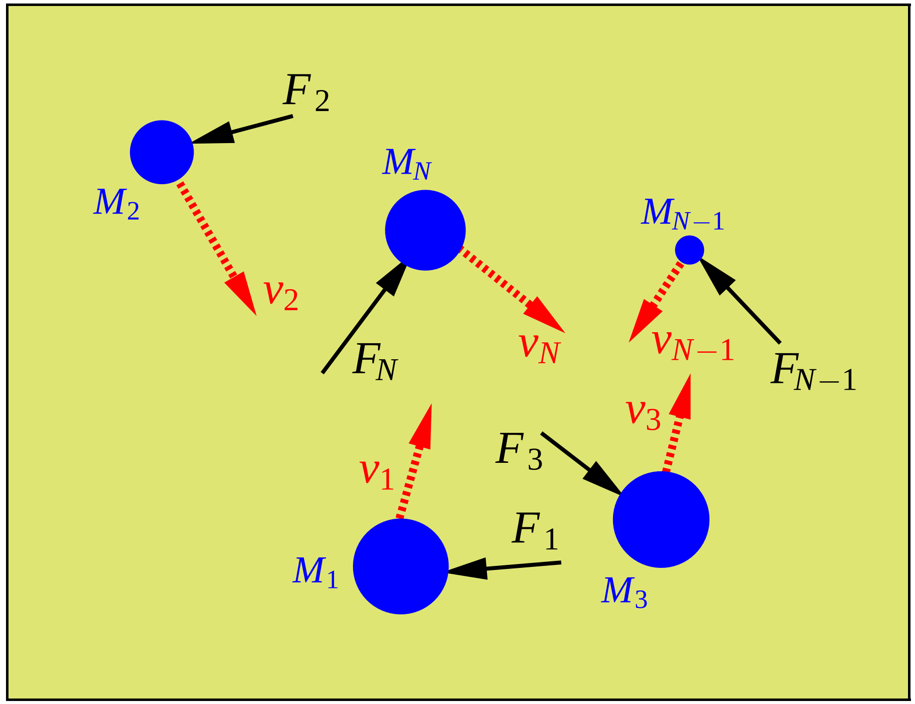
Generalization to N masses.



$$\mathcal{E}_{\textbf{motion}} = \frac{1}{4} \sum_{i,j \in \{1,2,\ldots,N\}} \frac{M_i \, M_j}{M_1 + M_2 + \cdots + M_N} ||v_i - v_j||^2.$$

Motion energy

With external forces.



$$\mathscr{E}_{\textbf{motion}} = \frac{1}{4} \sum_{i,j \in \{1,2,\ldots,N\}} \frac{M_i\, M_j}{M_1 + M_2 + \cdots + M_N} ||v_i - v_j||^2.$$

(KFL) $\sum_{i \in \{1,2,\ldots,N\}} F_i = 0 \quad \Rightarrow \quad \frac{d}{dt}\mathscr{E}_{\textbf{motion}} = \sum_{i \in \{1,2,\ldots,N\}} F_i^\top v_i.$

Motion energy

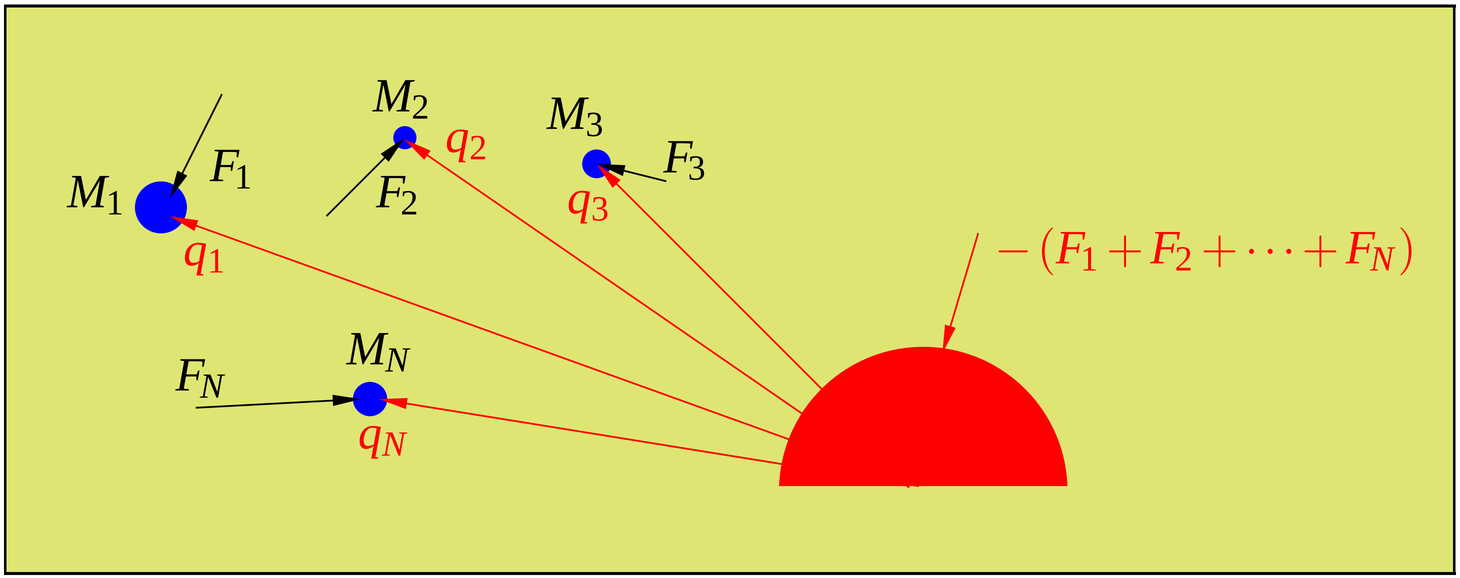
$$\mathscr{E}_{\textbf{motion}} = \frac{1}{4} \sum_{i,j \in \{1,2,\ldots,N\}} \frac{M_i \, M_j}{M_1 + M_2 + \cdots + M_N} ||v_i - v_j||^2.$$

Distinct from the classical expression of the kinetic energy,

$$\mathscr{E}_{\textbf{kinetic}} = \frac{1}{2} \sum_{i \in \{1,2,\ldots,N\}} M_i \, ||v_i||^2.$$

Motion energy

Reconciliation: $M_{N+1} = \infty, F_{N+1} = -(F_1 + F_2 + \cdots + F_N)$,



measure velocities w.r.t. this infinite mass (‘ground’), then

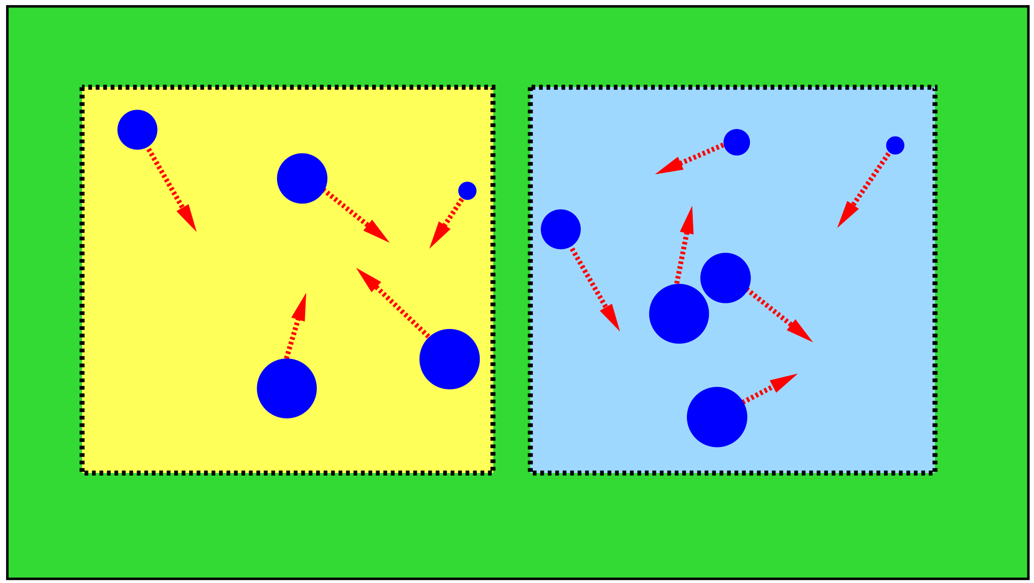
$$\frac{1}{4} \sum_{i,j \in \{1,2,\ldots,N,N+1\}} \frac{M_i \, M_j}{M_1 + M_2 + \cdots + M_N + M_{N+1}} ||v_i - v_j||^2$$

$$\longrightarrow \quad \frac{1}{2} \sum_{i \in \{1,2,\ldots,N\}} M_i \, ||v_i||^2.$$

Energy as an extensive quantity

Motion energy

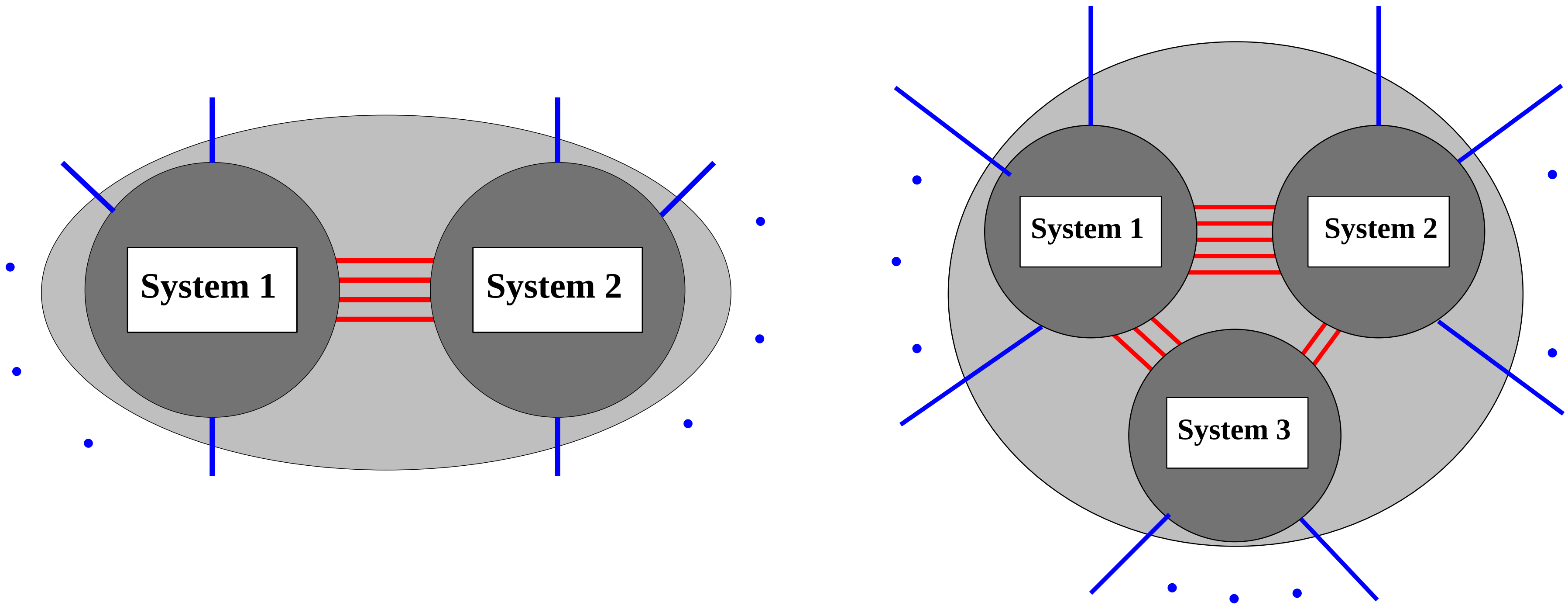
Motion energy is not an extensive quantity, it is not additive.



Total motion energy $\neq$ sum of the parts.

Concluding remarks

Energy transfer



One cannot speak about

"the energy transferred from system 1 to system 2"
or *"from the environment to system 1"*,

unless the relevant terminals form a port.

Power and energy are not ‘local’, they involve ‘action at a distance’.

Ports and terminals

Terminals are for interconnection,

ports are for energy transfer.

Copies of the lecture frames are available from/at

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http://www.esat.kuleuven.be/∼jwillems

Thank you

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