# INCREMENTAL MULTIPLE ERROR FILTERED-X LMS FOR NODE-SPECIFIC ACTIVE NOISE CONTROL OVER WIRELESS ACOUSTIC SENSOR NETWORKS

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## ABSTRACT

We propose an adaptive distributed algorithm to solve a nodespecific Active Noise Control (ANC) problem. In this novel ANC problem, the nodes estimate different but overlapping ANC filters in order to generate secondary signals that cancel a primary noise source as it impinges on their microphones. Different sets of nodes follow a cyclic mode of cooperation in order to implement several coupled Multiple Error Filtered-X Least Mean Squares algorithms, each responsible for the estimation of part of the different nodespecific ANC filters. The proposed algorithm outperforms the noncooperative strategies and achieves the same steady-state noise reduction as a centralized solution that processes all the signals in the network. Finally, computer simulations are provided to illustrate the effectiveness of the proposed algorithm.

*Index Terms*— Distributed node-specific parameter estimation, wireless sensor networks, active noise control.

## 1. INTRODUCTION

To solve signal processing problems over wireless sensor networks (WSNs), several distributed estimation techniques have been proposed [1]-[19]. Initially, most of these techniques have been applied to networks where all nodes cooperate with each other to estimate the same network-wide signal or parameter (e.g. [1]-[5]). More recently, due to the heterogeneity of the devices that form networks in today's digital age, there is a growing interest in designing distributed estimation techniques that can be applied over multi-task networks where the devices are interested in solving different but overlapping node-specific estimation problems.

In the context of node-specific estimation problems over WSNs, two major groups of works can be distinguished. The first group considers distributed algorithms that allow to estimate samples of node-specific desired signals sharing a common latent signal subspace. These distributed algorithms apply compressive filter-andsum operations on the sensor signals in WSNs with a fully-connected topology [6], a tree topology [7] and combinations thereof [8] allowing for a single-shot estimation of every signal sample at each node. The second group considers distributed algorithms under which nodes of an ad-hoc WSN cooperate to simultaneously solve different but related parameter estimation problems. In [9] the authors have proposed a diffusion-based algorithm with spatial regularizers that leverage an a priori knowledge on the relationship between the node-specific parameter estimation (NSPE) problems to facilitate the cooperation between nodes with similar estimation interests. Although this cooperation allows to achieve superior performance compared to the non-cooperative approach, it yields biased estimates. The authors in [10]-[12] have proposed incremental and diffusion strategies that let the nodes obtain asymptotically unbiased estimates in a NSPE problem where the nodes have a-priori known node-specific interests. To solve this NSPE problem and simultaneously learn the relationship between the NSPE problems of neighboring nodes, a handful of works have also proposed unsupervised diffusion strategies with adaptive combination techniques determined through different multi-task clustering techniques [13], [14].

Besides solving generic adaptive learning and optimization problem over multi-task networks, there is an increasing number of works addressing the design of distributed algorithms that leverage the cooperation among nodes in a so-called wireless acoustic sensor network (WASN) for speech and audio applications [15]-[20]. In this paper, we focus on the application of active noise control (ANC) [21], for which recently also several distributed algorithms have been proposed [17]-[19], with applications in the automotive and aeronautic industry in order to improve auditory comfort of passengers. A distributed ANC system consists of a multitude of nodes, each equipped with a set of microphones that record a primary noise source and a set of loudspeakers that act on the environment by emitting signals aimed at canceling the recorded noise source. However, to the authors knowledge, when addressing the design of such distributed ANC systems, all existing approaches assume that all the nodes are interested in estimating the same network-wide ANC filter, which does not generally hold due to node-specific acoustical coupling among the nodes. Moreover, none of the proposed techniques are scalable with the network size. To overcome these limitations, we state the novel node-specific ANC problem where the nodes can be interested in estimating different but overlapping ANC filters. We design an incremental Multiple Error Filtered-X Least Mean Squares (MEFxLMS) algorithm for node-specific ANC that achieves the same performance as the corresponding centralized solution [21]. To do so, the nodes run different but coupled MEFxLMS algorithms under an incremental mode of cooperation. Finally, computer simulations are provided to illustrate the effectiveness of the proposed algorithm.

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**Fig. 1.** WASN with K nodes. A link between the nodes indicates that they are acoustically coupled.

The following notation is used throughout the paper. We use boldface letters for random variables and normal fonts for deterministic quantities. Capital letters refer to matrices and small letters refer to both vectors and scalars. The notation  $(\cdot)^H$  and  $E\{\cdot\}$  stand for the Hermitian transposition and the expectation operator, respectively. Finally,  $\|\mathbf{x}\|$  equals the Euclidian norm of  $\mathbf{x}$  and  $0_{L\times M}$  denotes the  $L \times M$  zero matrix.

#### 2. PROBLEM FORMULATION

We consider a WASN consisting of K nodes deployed over some region. As shown in Fig. 1, each node is equipped with a single microphone and a loudspeaker<sup>1</sup>. Moreover, the nodes have P given reference signals that are correlated with a primary noise source. Without loss of generality, we assume that these reference signals are the same at all nodes.

The goal of the ANC implemented at node k, is to emit a filtered version of the P available reference signals and to cancel a primary noise source as it impinges on its microphone. As a result, considering that the channel between the loudspeaker of node  $\ell$  and the microphone of node k is modeled as an L-th order FIR filter, i.e.,  $h_{\ell k} = \operatorname{col}\{h_{\ell k}(l)\}_{l=1}^{L} \in \mathbb{C}^{L \times 1}$ , at time instant i the error signal  $e_k(i)$  measured at the microphone of node k is described as

$$\mathbf{e}_{k}(i) = \mathbf{d}_{k}(i) + \sum_{\ell \in \mathcal{I}_{k}} \sum_{l=1}^{L} h_{\ell k}(l) \sum_{p=1}^{P} \mathbf{x}_{p,i-l+1} w_{\ell p,i-l+1}$$
(1)

where

 d<sub>k</sub>(i) denotes the primary noise source as it impinges on the microphone of node k,

- $w_{\ell p,i} \in \mathcal{C}^{M \times 1}$  denotes filter of M coefficients applied by node  $\ell$  to the p-th reference signal at time instant i,
- $\mathbf{x}_{p,i} = [\mathbf{x}_p(i) \cdots \mathbf{x}_p(i-M+1)] \in \mathcal{C}^{1 \times M}$  with  $\mathbf{x}_p(i)$  denoting the *p*-th reference signal at time instant *i*,
- $\mathcal{I}_k$  denotes an ordered set of indices associated with the nodes whose loudspeaker is acoustically coupled with the microphone of node k, i.e., the nodes whose emitted signals can be observed with significant power at the microphone of node k.

Considering that the secondary channels  $\{h_{\ell k}\}_{\ell \in \mathcal{I}_k}$  are known or estimated by node k in a calibration phase [22], the ordered sets  $\{\mathcal{I}_k\}$  can also assumed to be known or estimated. Given these estimates or prior knowledge, the objective of the nodes is to cooperate in order to process the data set  $\{\mathbf{e}_k(i)\}_{k=1}^K$  and find the ANC filters  $\{\{w_{kp,i}\}_{p=1}^P\}_{k=1}^K$  that minimize

$$J_{\text{glob}}\left(\left\{\{w_{kp,i}\}_{p=1}^{P}\right\}_{k=1}^{K}\right) = \sum_{k=1}^{K} E\left\{\mathbf{e}_{k}^{2}(i)\right\}$$
(2)

with  $\mathbf{e}_k$  defined in (1). All the works addressing the design of distributed ANC systems [17]-[19] assume that all the nodes are interested in cooperating to estimate the same network-wide ANC filter, i.e.,  $r_i = \operatorname{col}\left\{\{w_{\ell p,i}\}_{\ell=1}^{P}\right\}_{\ell=1}^{k}$ . Instead, in this paper we consider a more general setting where the nodes are interested in estimating different but related ANC filters. In particular, according to the observation model (1) of the considered node-specific ANC system, each node k is interested in estimating the filter  $r_{k,i} = \operatorname{col}\{w_{\ell,i}\}_{\ell \in \mathcal{I}_k}$  where

$$w_{\ell,i} = \operatorname{col}\{w_{\ell p,i}\}_{p=1}^{P}.$$
(3)

Since the sets  $\{\mathcal{I}_k\}_{k=1}^K$  are not disjoint, note that the ANC filters  $r_{k,i}$  estimated by different nodes can be partially overlapping. Indeed, the ANC filter estimated by node k will be overlapping with the ANC filter of a node  $\ell$  as long as  $\mathcal{I}_k \cap \mathcal{I}_\ell \neq \emptyset$ . Moreover, since the sets  $\mathcal{I}_k$  differ from node to node, notice that the ANC filters of two different nodes can be arbitrarily different. Despite this fact, a distributed algorithm can be proposed to let the nodes cooperate and achieve the same performance as a centralized approach.

## 3. DISTRIBUTED NODE-SPECIFIC ANC

In this section, first we provide a centralized solution for the nodespecific ANC problem provided in (2), and then we develop an incremental distributed algorithm that converges to this centralized solution. As it is often assumed in the literature [21], we assume that the different ANC filters are time-invariant. In particular, we consider that  $w_{\ell,i} = w_{\ell}$  in (3). Note that this assumption is approximately satisfied if the coefficients of the ANC filters change or adapt slowly as compared to the timescale of the system to be controlled, i.e., the secondary channels  $\{h_{\ell k}\}_{\ell \in \mathcal{I}_k}$  and the channel between the primary noise source and the microphones.

## **3.1.** Centralized solution

First, under the assumed time invariance of the ANC filters, note that the error signal measured by the microphone of node k is

$$\mathbf{e}_{k}(i) = \mathbf{d}_{k}(i) + \sum_{\ell \in \mathcal{I}_{k}} h_{\ell k}^{H} \mathbf{X}_{i} w_{\ell}$$
(4)

where  $w_{\ell}$  equals (3) with the time-dependence *i* removed and where

$$\mathbf{X}_{i} = \begin{bmatrix} \mathbf{X}_{1,i} \, \mathbf{X}_{2,i} \cdots \, \mathbf{X}_{P,i} \end{bmatrix}$$
(5)

with  $\mathbf{X}_{p,i} = \operatorname{col}\{\mathbf{x}_{p,i-l+1}\}_{l=1}^{L} \in \mathbb{C}^{L \times M}$ . Hence, by substituting (4) into (2), the solution of the considered distributed node-specific ANC requires the optimization of

$$J_{\text{glob}}\left(\left\{w_{\ell}\right\}_{\ell=1}^{K}\right) = \sum_{k=1}^{K} E\left\{\left|\mathbf{d}_{k}(i) + \sum_{\ell \in \mathcal{I}_{k}} h_{\ell k}^{H} \mathbf{X}_{i} w_{\ell}\right|^{2}\right\}$$
(6)

with respect to multiple vector variables, i.e.,  $\{w_\ell\}_{\ell=1}^K$ . If we now gather all the variables associated with the different ANC filters into the following augmented vector

$$\tilde{w} = \left\{ w_{\ell} \right\}_{\ell=1}^{K} \quad (\widetilde{M} \times 1) \tag{7}$$

<sup>&</sup>lt;sup>1</sup>For the sake of clarity, we have assumed single channel nodes, though the derivations can be easily extended to multi-channel nodes that have N microphones and J loudspeakers with N, J > 1 and  $N \ge J$  [21].

where  $\widetilde{M} = KPM$ , from (4) we can easily verify that

$$\mathbf{e}_k(i) = \mathbf{d}_k(i) + \widetilde{\mathbf{u}}_{k,i}\widetilde{w} \tag{8}$$

where

$$\widetilde{\mathbf{u}}_{k,i} = \left[\mathbb{1}_{\{1\in\mathcal{I}_k\}}h_{1k}^H\mathbf{X}_i \ \mathbb{1}_{\{2\in\mathcal{I}_k\}}h_{2k}^H\mathbf{X}_i \ \cdots \ \mathbb{1}_{\{K\in\mathcal{I}_k\}}h_{Kk}^H\mathbf{X}_i\right]$$
(9)

with  $\mathbb{1}_{\{\mathcal{X}\in\mathcal{A}\}}$  denoting an indicator function that equals 1 if  $\mathcal{X}\in\mathcal{A}$  or 0 otherwise. Thus, the node-specific ANC problem in (6) can be cast as

$$\widehat{\tilde{w}} = \operatorname*{argmin}_{\tilde{w}} \{ J_{glob}(\tilde{w}) \} = \operatorname*{argmin}_{\tilde{w}} \sum_{k=1}^{K} E\left\{ \left| \mathbf{d}_{k}(i) + \widetilde{\mathbf{u}}_{k,i} \widetilde{w} \right|^{2} \right\}$$
(10)

The centralized solution  $\hat{\tilde{w}}$  is given by the normal equations [23]

$$\left(\sum_{k=1}^{N} R_{\widetilde{u}_{k,i}}\right) \cdot \widehat{\widetilde{w}} = -\sum_{k=1}^{N} r_{\widetilde{u}_{k,i}d_{k,i}}$$
(11)

with  $R_{\tilde{u}_{k,i}} = E\{\tilde{\mathbf{u}}_{k,i}^{H}\tilde{\mathbf{u}}_{k,i}\}\)$  and  $r_{\tilde{u}_{k,i}d_{k,i}} = E\{\tilde{\mathbf{u}}_{k,i}^{H}\mathbf{d}_{k,i}\}\)$ . However, when computing this centralized solution with an adaptive filter, e.g., based on filtered-X LMS [21], each node would have to transmit its error signal to a central device, where the filters are computed and then transmitted to the nodes. This is not robust, as it introduces a single point of failure, and moreover, it is not scalable with the network size since it requires the inversion of a square matrix whose dimension is proportional to the number of nodes K. To alleviate this prohibitive computational cost and communication requirement, a distributed algorithm is proposed in the next section.

### 3.2. Distributed algorithm

Similar to [17], to design a distributed algorithm for the estimation of  $\hat{w}$ , our starting point is the traditional steepest-descent algorithm. This algorithm allows to iteratively estimate  $\tilde{w}$  by splitting the update from  $\tilde{w}^{(i)}$  to  $\tilde{w}^{(i+1)}$  into partial updates across the network, where  $\tilde{w}^{(i)} = \operatorname{col}\{w_k^{(i)}\}_{k=1}^K$  denotes the estimate of  $\tilde{w}$  at time instant *i*. In particular, taking into account that our global cost function  $J_{\text{glob}}(\tilde{w})$  is expressed as the sum of *K* local cost functions  $\{J_k(\tilde{w})\}_{k=1}^K$  with

$$J_k(\tilde{w}) = E\left\{\mathbf{e}_k^2(i)\right\} = E\left\{\left|\mathbf{d}_k(i) + \widetilde{\mathbf{u}}_{k,i}\tilde{w}\right|^2\right\}$$
(12)

and  $\widetilde{\mathbf{u}}_{k,i}$  defined in (9), at each instant *i* a steepest descent for (10) should perform the following step for any node  $k \in \{1, 2, \ldots, K\}$  and some initialization  $\widetilde{\psi}_{K}^{(0)}$ 

$$\tilde{\psi}_{k}^{(i)} = \tilde{\psi}_{(k-1)}^{(i)} - \mu_{k} \left[ \nabla J_{k}(\tilde{w}^{(i)}) \right]^{H}$$
(13)

where 
$$\widehat{w}^{(i)} = \widehat{\psi}_{K}^{(i)}$$
 and  
 $\left[\nabla J_{k}(\widetilde{w}^{(i)})\right]^{H} = E\left\{\widetilde{\mathbf{u}}_{k,i}^{H}\mathbf{e}_{k}(i)\right\} = E\left\{\widetilde{\mathbf{u}}_{k,i}^{H}\left[\mathbf{d}_{k}(i) + \widetilde{\mathbf{u}}_{k,i}\widetilde{w}^{(i)}\right]\right\}$ 
(14)

with  $\tilde{\psi}_k^{(i)} = \operatorname{col}\{\psi_{k\ell}^{(i)}\}_{\ell=1}^K$  denoting a local estimate of  $\tilde{w}$  at node k and time instant i,  $\tilde{\psi}_K^{(0)}$  equal to some random guess of  $\tilde{w}$ ,  $\mu_k$  denoting a suitably chosen positive step-size and

$$\tilde{\psi}_{(k-1)}^{(i)} = \begin{cases} \tilde{\psi}_{K}^{(i-1)} & \text{if } k = 1, \\ \tilde{\psi}_{k-1}^{(i)} & \text{otherwise.} \end{cases}$$
(15)



**Fig. 2.** Structure of the I-NS ANC algorithm for the estimation of the ANC filter  $w_{\ell}$  that generates the secondary signal at node  $\ell$ .

It is well known that the previous steepest-descent approach converges to the centralized solution  $\hat{w}$ , i.e.,  $\lim_{i\to\infty} \tilde{\psi}_k^{(i)} = \hat{w}$  if each step-size satisfies  $0 < \mu_k < 2/\lambda_{\max}$  with  $\lambda_{\max}$  equal to the largest eigenvalue of the invertible matrix  $\sum_{k=1}^N R_{\tilde{u}_k}$  [23]. However, note that its implementation requires the knowledge of  $E\left\{\tilde{\mathbf{u}}_{k,i}^H \mathbf{e}_k(i)\right\}$ , which depends on the statistics  $R_{\tilde{u}_k}$  and  $r_{\tilde{u}_k,d_k}$ . Since these statistics are not generally available, a widely-used approach [17] consists in using the following instantaneous approximations of the gradient

$$\left[\nabla J_k(\tilde{w}^{(i)})\right]^H \approx \tilde{u}_{k,i}^H e_k(i) = \tilde{u}_{k,i}^H \left[d_k(i) + \tilde{u}_{k,i}\tilde{w}^{(i)}\right]$$
(16)

where  $e_k(i)$  is the error signal measured at the microphone of node k when each node  $\ell$  generates the following secondary source signal

$$y_{\ell,i} = [x_{1,i} \ x_{2,i} \ \cdots \ x_{P,i}] \ w_{\ell}^{(i)}. \tag{17}$$

The previous approximation enables the network to respond to timevariations in the underlying signal statistics. Nonetheless, note that its implementation, in particular, the generation of the secondary source signals, requires each node k to have access to the global information, i.e.,  $\tilde{w}^{(i)}$ , which is only computed at node K once all the nodes have executed the adaptation step in (13). To overcome this, similarly to the incremental gradient technique ([3], [17], [24] and [25]), we consider that each node k, at time instant i, generates the secondary source signal by using its local estimate

$$y_{\ell,i} = [x_{1,i} \ x_{2,i} \ \cdots \ x_{P,i}] \psi_{\ell\ell}^{(i-1)}$$
(18)

to evaluate the instantaneous approximation of  $\nabla J_k(\cdot)$  at the local estimate  $\operatorname{col}\{\tilde{\psi}_{\ell\ell}^{(i-1)}\}_{\ell=1}^K$  instead of  $\tilde{w}^{(i)} = \tilde{\psi}_K^{(i-1)}$ . Thus, in the resulting algorithm, at instant *i* node *k* executes the following steps:

$$\begin{cases} \text{Transmission step: } y_{k,i} = [x_{1,i} \ x_{2,i} \ \cdots \ x_{P,i}] \ \psi_{kk}^{(i-1)}, \\ \text{Adaptation step: } \widetilde{\psi}_{k}^{(i)} = \widetilde{\psi}_{(k-1)}^{(i)} - \mu_k \widetilde{u}_{k,i}^H e_k(i). \end{cases}$$
(19)

In the cyclic cooperation established by the previous algorithm, at each time instant *i* each node *k* only needs to transmits the local estimate  $\tilde{\psi}_k^{(i)}$  to one neighbor. Although this solution is fully distributed, it is still non-scalable with respect to both communication requirement and computational cost since the dimension of  $\tilde{\psi}_k^{(i)}$  equals KPM, which depends on the total number K of nodes. This issue will be addressed in the following.

Due to the structure of  $\tilde{u}_{k,i}$  defined in (9), only  $|\mathcal{I}_k|$  sub-vectors of  $\tilde{\psi}_k^{(i)}$  are updated when a specific node k performs the Adaptation step at time instant *i* (see (19)). In particular, according to (9) and (19), only the sub-vectors associated with the local estimates of  $\{w_\ell\}_{\ell \in \mathcal{I}_k}$  at node k and time instant *i*, denoted as  $\{\psi_{k\ell}^{(i)}\}_{\ell \in \mathcal{I}_k}$ , are updated based on the local estimates  $\{\psi_{\ell(k)}^{(i)}\}_{\ell \in \mathcal{I}_k}$  where

$$\psi_{f_{\ell}(k)\ell}^{(i)} = \begin{cases} \psi_{c_{\ell}\ell}^{(i-1)} & \text{if } \mathcal{C}_{k\ell} = \emptyset, \\ \psi_{\max\{\mathcal{C}_{k\ell}\}\ell}^{(i)} & \text{otherwise,} \end{cases}$$
(20)

with  $C_{k\ell} = \{j \in C_{\ell} : j < k\}, c_{\ell} = \max\{C_{\ell}\} \text{ and } C_{\ell} = \{k : \ell \in \mathcal{I}_k\}$  which is not necessarily equal to  $\mathcal{I}_k$  unless there exists acoustical reciprocity. Thus, after defining  $\psi_k^{(i+1)} = \operatorname{col}\{\psi_{k\ell}^{(i+1)}\}_{\ell \in \mathcal{I}_k}, \psi_{(k-1)}^{(i+1)} = \operatorname{col}\{\psi_{f_{\ell}(k)\ell}^{(i+1)}\}_{\ell \in \mathcal{I}_k} \text{ and } u_{k,i}^H = \operatorname{col}\{X_i^H h_{\ell k}\}_{\ell \in \mathcal{I}_k}, \text{ in (19)}$  we obtain the incremental MEFxLMS for node-specific ANC (I-NS ANC), which is summarized as follows

## Incremental node-specific ANC (I-NS ANC)

- Start with some random guess  $\left\{\psi_{c_kk}^{(0)}\right\}_{k=1}^{K}$ .
- At time instant i, for each k ∈ {1, 2, ..., K} collect one extra sample of the reference signals to build X<sub>i</sub> and execute
   1. Transmission step:

$$y_{k,i} = \begin{bmatrix} 1 & 0_{1 \times (L-1)} \end{bmatrix} X_i \psi_{kk}^{(i-1)}$$
2. Adaptation step:  

$$\psi_k^{(i)} = \psi_{(k-1)}^{(i)} - \mu_k u_{k,i}^H e_k(i)$$
(21)

Note that the I-NS ANC solves a total of K different but coupled optimization problems simultaneously and in a distributed fashion. To this end, K cyclic modes of cooperation are simultaneously established. As illustrated in Fig. 2, the nodes in  $C_{\ell}$  undertake a cyclic mode of cooperation whose goal is to solve one of the optimization problems, which consists in estimating the filter  $w_{\ell}$  generating the secondary source signal emitted by the loudspeaker of node k. Similarly to other existing distributed algorithms [17]-[19]as well as the algorithm for node-specific ANC described in (20), in the proposed algorithm the resulting estimates asymptotically converge in the mean to the centralized solution if the positive step-sizes  $\{\mu_k\}_{k=1}^{K}$  are sufficiently small. The details of the proof are omitted due to space constraints. Unlike any other distributed algorithm for ANC, it should also be noted that the I-NS ANC algorithm is scalable with the network size in terms of computational cost and communication requirements. Regarding the computational cost, at each time instant, each node k only needs to update  $|\mathcal{I}_k|$  vectors whose dimensions are independent of the number of nodes. Moreover, decreasing the communication requirements, at each time instant i, each node k only transmits the local estimates of  $|\mathcal{I}_k|$  ANC filters, whose dimensions again do not depend on the number of nodes.

### 4. SIMULATIONS

To illustrate the effectiveness of the proposed algorithm, we consider a node-specific ANC system formed by K = 5 randomly deployed nodes, each equipped with one microphone and one loudspeaker. The goal for each node is to cancel a primary Gaussian noise source (with zero mean and unit variance) as locally observed at its microphone, i.e., after the primary noise source has been filtered with a node-specific impulse response of 20 taps (this impulse response is unknown). To do so, at each time instant i the loudspeaker of each node will emit a filtered version of only one reference signal (P = 1). In this system, we have assumed that the filter  $w_k$  is of length M = 30 coefficients and that the reference signal of every node corresponds to the primary noise source before being filtered by the unknown acoustic channel between the unwanted noise and the microphones. We have also employed 20-tap FIR filters to model the secondary channels. Moreover, in a setting with acoustical reciprocity, we have assumed that the loudspeaker of a node is acoustically coupled with a subset of nodes in the network, i.e., its emitted secondary source signal can be measured at the microphones of other nodes. In particular, we have considered that  $C_1 = \{1, 2, 3, 5\}$ ,



**Fig. 3**. Evolution of Network NR and MSE for the non-cooperative ANC, the I-NS ANC algorithm and the centralized ANC.

 $C_3 = \{1, 3, 4\}, C_4 = \{2, 3, 4, 5\}, C_5 = C_2 = \{1, 2, 4, 5\}$  and  $C_k = \mathcal{I}_k$  for each node k (reciprocity of acoustical coupling holds).

In the simulations presented here, we compare the proposed I-NS ANC algorithm with the centralized solution provided in (11) and a non-cooperative ANC that is equivalent to the proposed I-NS ANC algorithm described in (21) when  $\psi_{(k-1)}^{(i+1)} = \psi_k^{(i)}$  and  $u_{k,i}^H =$  $X_i^H h_{kk}$  (see [17] for more details). To do this comparison, for each algorithm we have evaluated the instantaneous network Mean Square Error (MSE) and the network Noise Reduction (NR), defined as  $(1/K) \sum_{k=1}^{K} 10 \log_{10}[e_k^2(i)/d_k^2(i)]$ . Furthermore, since the I-NS ANC and the non-cooperative algorithms undertake  $|\mathcal{I}_k|$  and one updates of the estimate of  $w_k$  per time step, respectively, to have a fair comparison we have assumed that  $\mu_k^{\text{LNS ANC}} = \mu_k^{nc}/|\mathcal{I}_k| = 10^{-3}$ where  $\mu_k^{\text{LNS ANC}}$  and  $\mu_k^{nc}$  denote the step-size used by the I-NS ANC and the non-cooperative algorithms for the estimation of  $w_k$ , respectively. Under this assumption, Fig. 3 shows the temporal evolution of the network NR and MSE for the three aforementioned ANC algorithms. To generate each plot, the results have been averaged over 50 independent experiments. As expected, note that the I-NS ANC achieves the same steady-state NR and MSE as the centralized ANC. On the contrary, although the non-cooperative ANC initially cancels some primary noise, there is an instant from which it becomes unstable, and hence, shows a poor performance in the steady-state. As discussed in [17], this degradation occurs due to the absence of cooperation among the nodes when they are solving ANC problems that are indeed coupled through the secondary paths.

## 5. CONCLUSION

We have considered a node-specific ANC problem where the nodes simultaneously estimate different but overlapping filters to generate secondary source signals that cancel a primary noise source as it is observed at their microphones. To solve this problem, we have presented a technique based on several coupled MEFxLMS algorithms. The implementation of each MEFxLMS algorithm is undertaken by all the nodes that are acoustically coupled with a specific secondary source and its goal is the estimation of part of the different node-specific ANC filters. The proposed algorithm achieves the same steady-state noise reduction as the centralized solution and, unlike the existing distributed ANCs, is scalable with the network size. Computer simulations have shown the effectiveness of the algorithm.

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