Supplementary material for the article titled 'Fast Linear Least-Squares Method for Ultrasound Attenuation and Backscatter Estimation'

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ABSTRACT

The ultrasonic attenuation and backscatter coefficients of tissues are relevant acoustic parameters due to their wide range of clinical applications. In this paper, a linear least-squares method for the estimation of these coefficients in a homogeneous region of interest based on pulse-echo measurements is proposed. The method efficiently fits an ultrasound backscattered signal model to the measurements in both the frequency and depth dimension simultaneously at a low computational cost. It is demonstrated that the inclusion of depth information has a positive effect particularly on the accuracy of the estimated attenuation. The sensitivity of the attenuation and backscatter coefficients' estimates to several predefined parameters such as the window length, window overlap and usable bandwidth of the spectrum is also studied. Comparison of the proposed method with a benchmark approach based on dynamic programming highlights better performance of our method in estimating these coefficients, both in terms of accuracy and computation time. Further analysis of the computation time as a function of the predefined parameters indicates our method's potential to be used in real-time clinical settings.

The proposed linear least squares (LLS) method for the estimation of attenuation and backscatter coefficient was compared with the dynamic programming (DP) approach on several simulated datasets, varying the values of α_{true} and μ_{true}. In fig. 1, the results are provided for all the considered cases, showing relative estimation errors for α and μ as a function of the number of RF lines.

Further, a parameter sensitivity study was performed to analyze the influence of the parameters - window length, window overlap and usable bandwidth, on the accuracy of the obtained coefficients estimates. The corresponding results are shown in fig. 2, 3 and 4 for window length, window overlap percentage and usable bandwidth, respectively. In each plot, the relative error of the estimated coefficient is plotted against the respective parameter under study.

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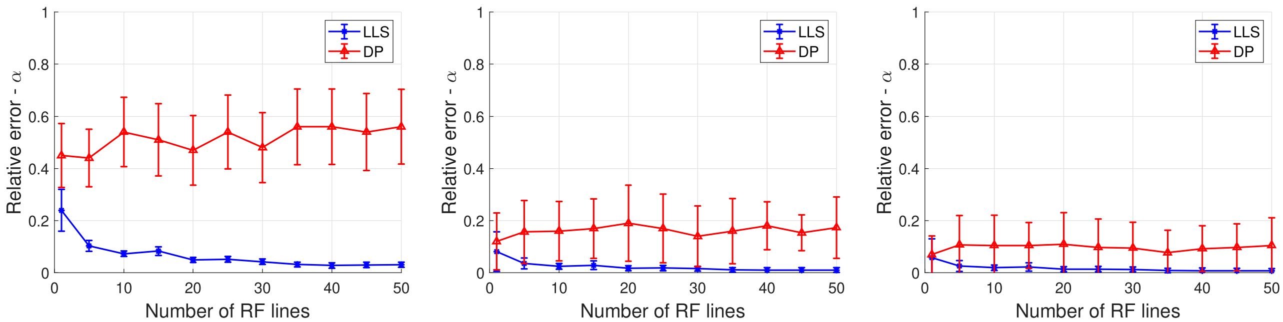
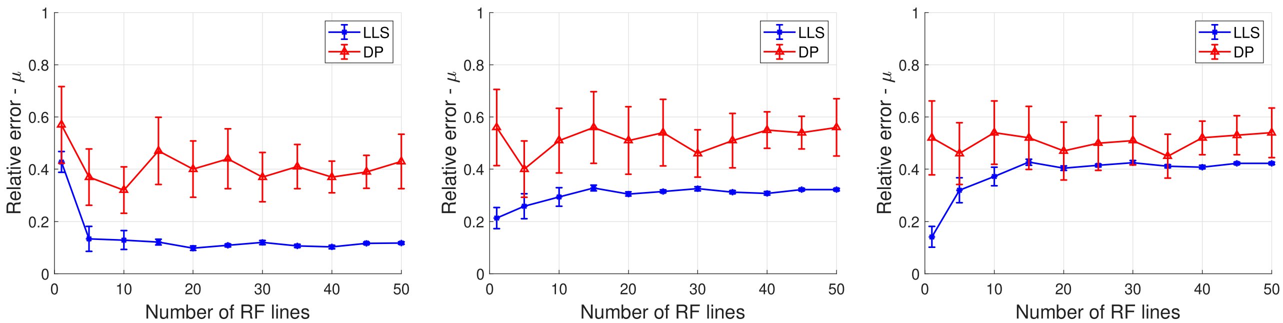
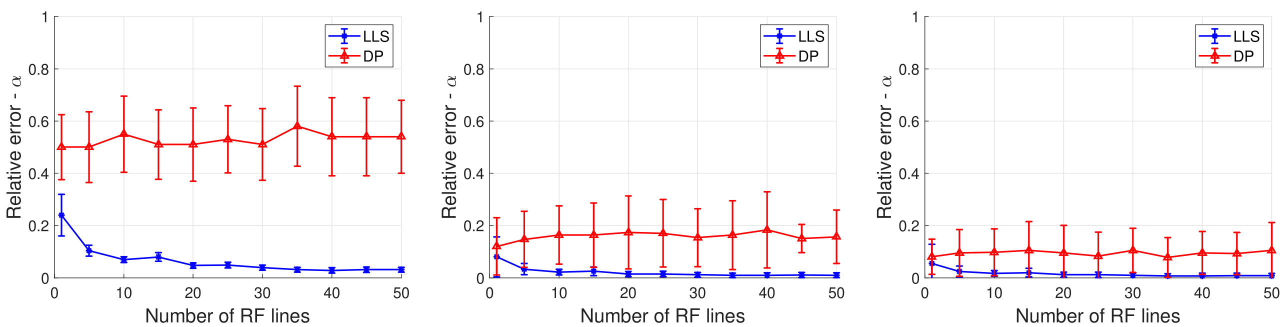


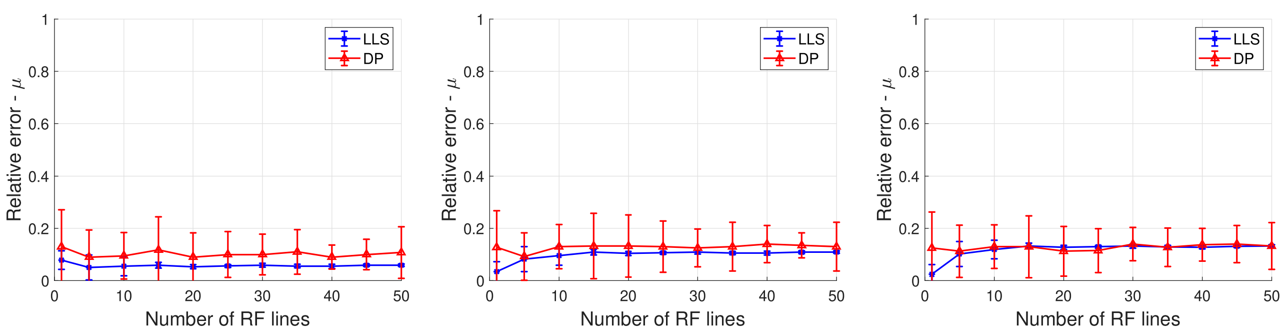
(a) α estimation error for $\mu_{true} = 0.5$, and left to right, $\alpha_{true} = 0.5, 1.5, 2$, respectively



(b) μ estimation error for $\mu_{true} = 0.5$, and left to right, $\alpha_{true} = 0.5, 1.5, 2$, respectively

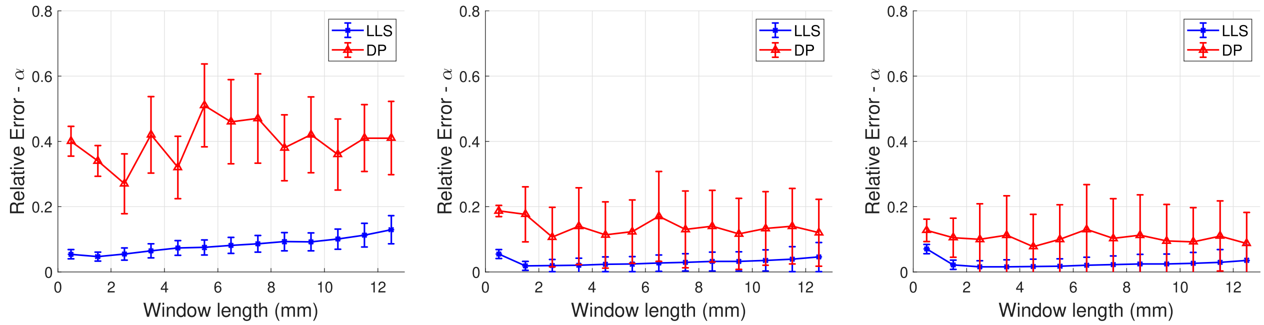


(c) α estimation error for $\mu_{true} = 2$, and left to right, $\alpha_{true} = 0.5, 1.5, 2$, respectively

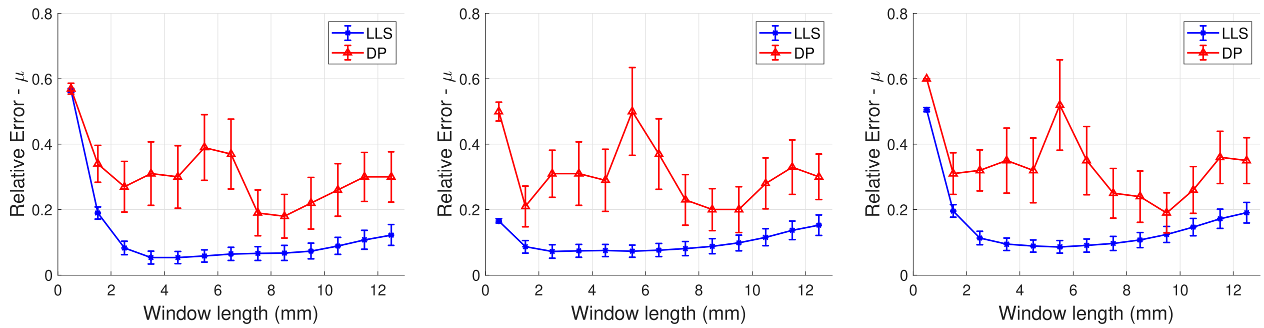


(d) μ estimation error for $\mu_{true} = 2$, and left to right, $\alpha_{true} = 0.5, 1.5, 2$, respectively

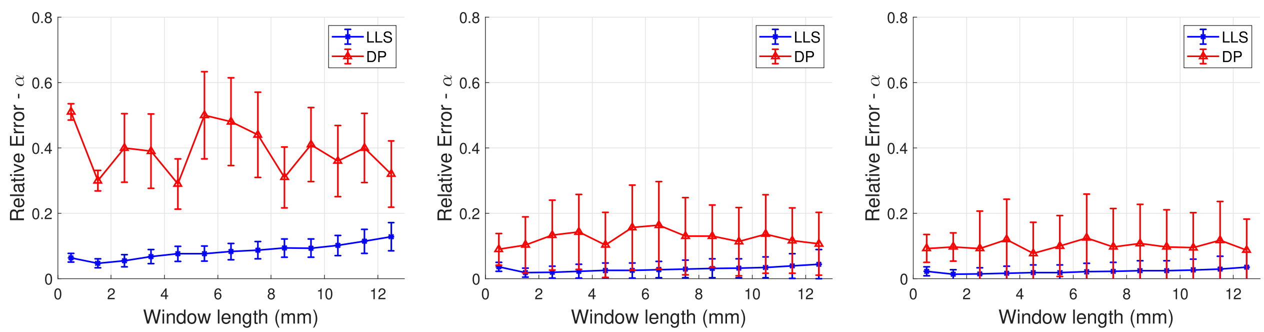
Figure 1: *The relative error of the α and μ estimates using* LLS *and* DP *is plotted as a function of the number of RF lines, used to make an observation sequence. The mean relative error is plotted, in which the error bars show the standard deviation of the errors when the experiment is repeated N_{rep} = 10 times. In each row, α_{true} is 0.5, 1.5 and 2 dB/cm/MHz, respectively from left to right. The first two and last two rows are for μ_{true} = 0.5 and 2, respectively. The window length chosen (4.92 mm) is twice the pulse-length of the transmitted pulse, the window overlap is 90% and the usable bandwidth (1.57 MHz) is equal to the bandwidth of the transmitted pulse.*



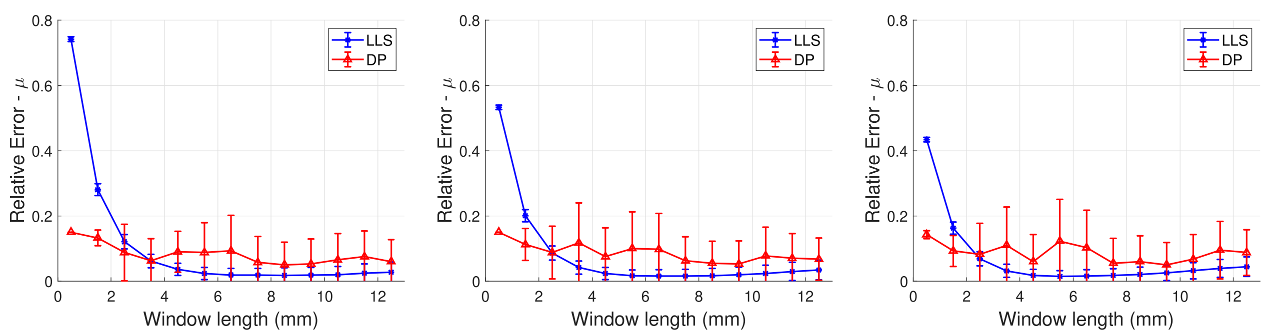
(a) α estimation error for $\mu_{true} = 0.5$, and left to right, $\alpha_{true} = 0.5, 1.5, 2$, respectively



(b) μ estimation error for $\mu_{true} = 0.5$, and left to right, $\alpha_{true} = 0.5, 1.5, 2$, respectively

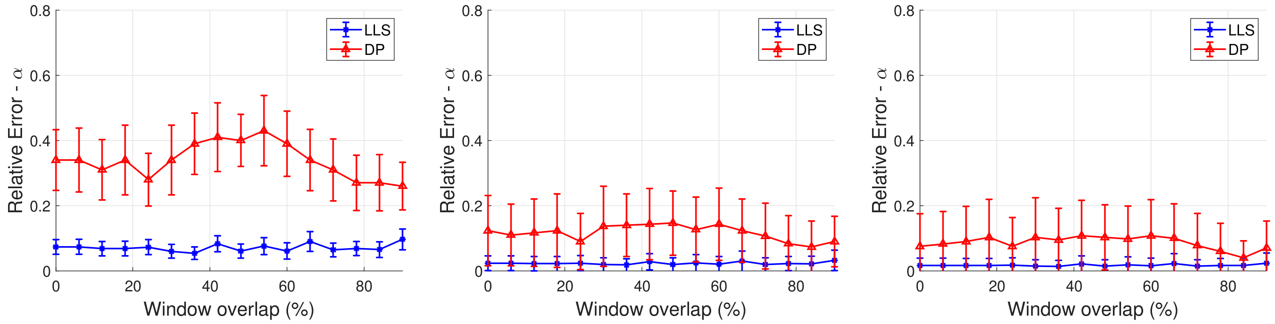


(c) α estimation error for $\mu_{true} = 2$, and left to right, $\alpha_{true} = 0.5, 1.5, 2$, respectively

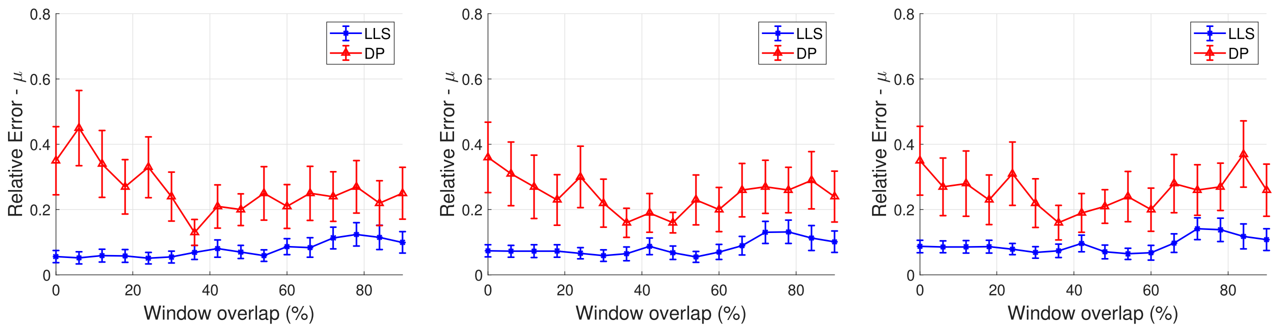


(d) μ estimation error for $\mu_{true} = 2$, and left to right, $\alpha_{true} = 0.5, 1.5, 2$, respectively

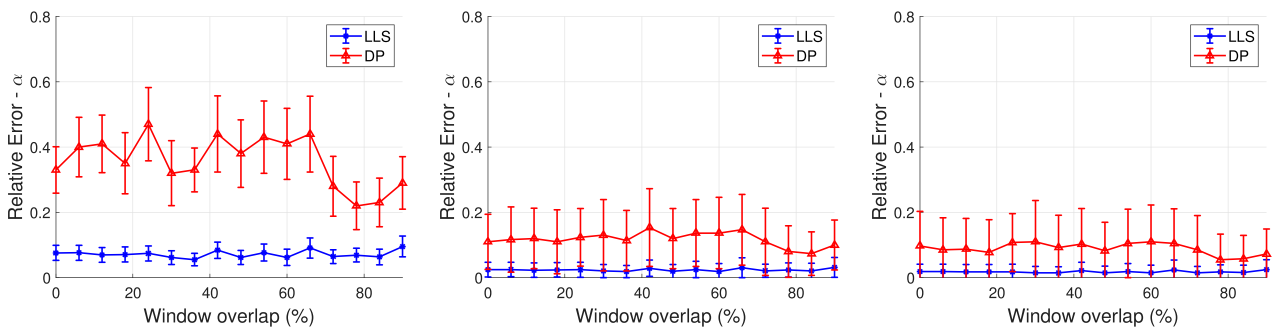
Figure 2: *The relative error of the α and μ estimates, where $N_{RF} = 50$, using* LLS *and* DP *is plotted against the window length. The mean relative error is plotted, in which the error bars show the standard deviation of the errors when the experiment is repeated $N_{rep} = 10$ times. In each row, α_{true} is 0.5, 1.5 and 2 dB/cm/MHz, respectively from left to right. The first two and last two rows are for $\mu_{true} = 0.5$ and 2, respectively. The window overlap is 90% and the usable bandwidth (1.57 MHz) is equal to the bandwidth of the transmitted pulse.*



(a) α estimation error for $\mu_{true} = 0.5$, and left to right, $\alpha_{true} = 0.5, 1.5, 2$, respectively



(b) μ estimation error for $\mu_{true} = 0.5$, and left to right, $\alpha_{true} = 0.5, 1.5, 2$, respectively



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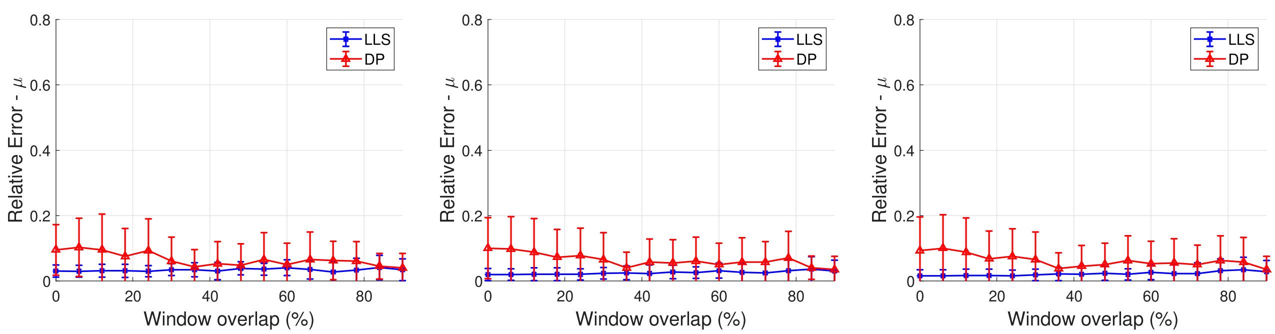
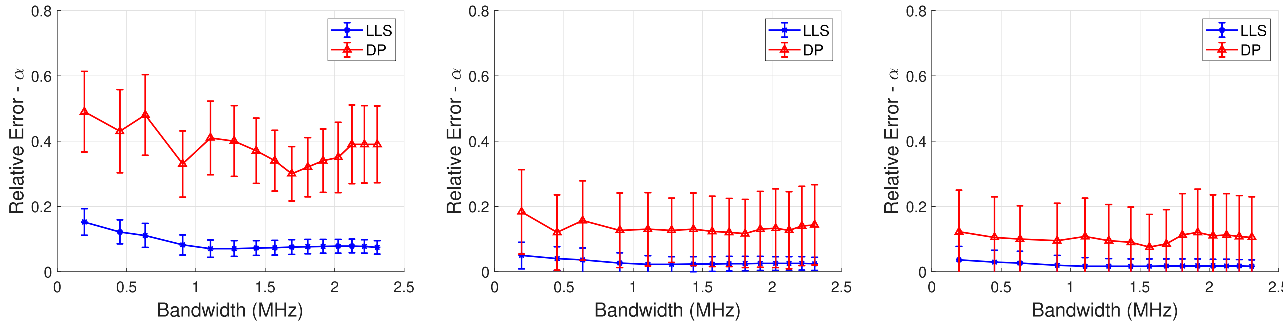
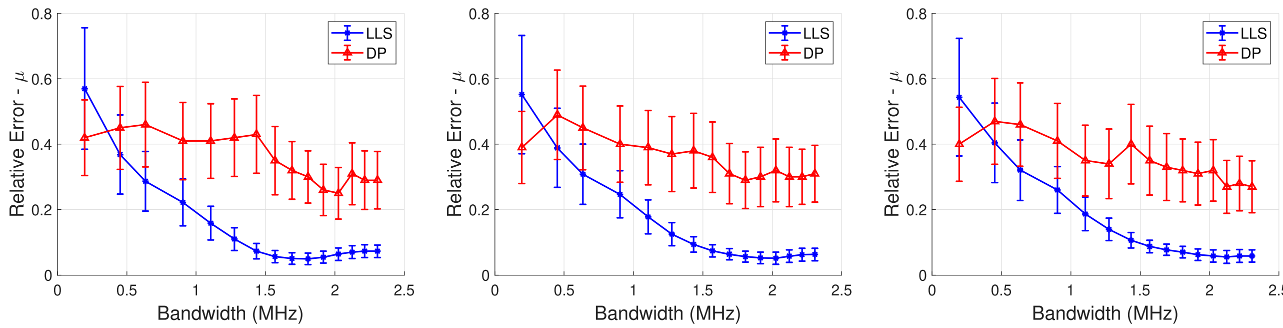


(d) μ estimation error for $\mu_{true} = 2$, and left to right, $\alpha_{true} = 0.5, 1.5, 2$, respectively

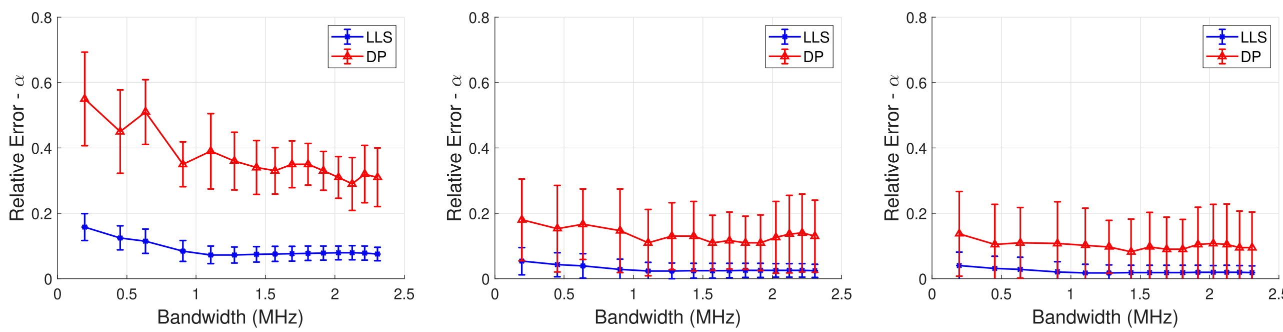
Figure 3: *The relative error of the α and μ estimates, where N_{RF} = 50, using* LLS *and* DP *is plotted against the window overlap percentage. The mean relative error is plotted, in which the error bars show the standard deviation of the errors when the experiment is repeated N_{rep} = 10 times. In each row, α_{true} is 0.5, 1.5 and 2 dB/cm/MHz, respectively from left to right. The first two and last two rows are for μ_{true} = 0.5 and 2, respectively. The window length (4.92 mm) is twice the pulse-length of the transmitted pulse and the usable bandwidth (1.57 MHz) is equal to the bandwidth of the transmitted pulse.*



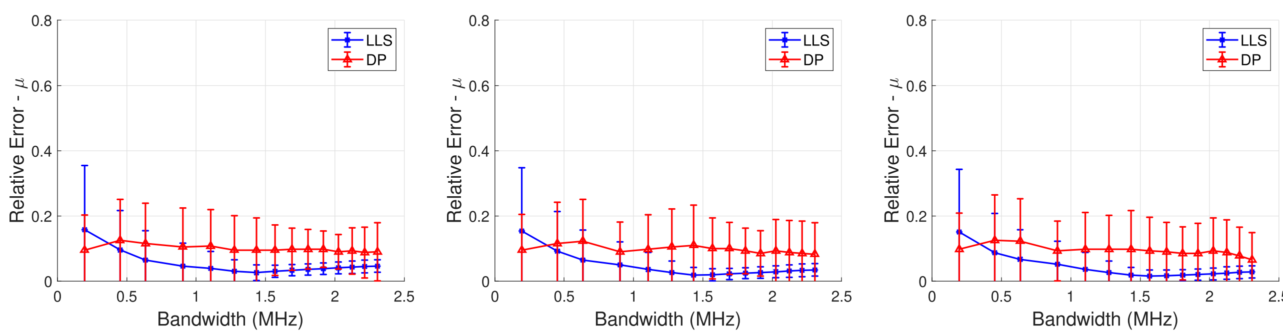
(a) α estimation error for $\mu_{true} = 0.5$, and left to right, $\alpha_{true} = 0.5, 1.5, 2$, respectively



(b) μ estimation error for $\mu_{true} = 0.5$, and left to right, $\alpha_{true} = 0.5, 1.5, 2$, respectively



(c) α estimation error for $\mu_{true} = 2$, and left to right, $\alpha_{true} = 0.5, 1.5, 2$, respectively



(d) μ estimation error for $\mu_{true} = 2$, and left to right, $\alpha_{true} = 0.5, 1.5, 2$, respectively

Figure 4: *The relative error of the α and μ estimates, where N_{RF} = 50, using* LLS *and* DP *is plotted against the usable bandwidth. The mean relative error is plotted, in which the error bars show the standard deviation of the errors when the experiment is repeated N_{rep} = 10 times. In each row, α_{true} is 0.5, 1.5 and 2 dB/cm/MHz, respectively from left to right. The first two and last two rows are for μ_{true} = 0.5 and 2, respectively. The window length (4.92 mm) is twice the pulse-length of the transmitted pulse and the window overlap is 90%.*