Introduction: Quantitative Ultrasound (QUS) refers to (signal processing) methodologies that enable the quantification of the acoustic properties of the propagation medium, such as its attenuation coefficient. In particular, the attenuation coefficient of tissues is relevant due to its wide range of clinical applications and because it allows performance of adaptive time gain compensation to enhance imaging. Earlier attempts were made to estimate the attenuation coefficient from uniform homogeneous media, which includes methods such as spectral shift, spectral difference [1]-[3] and methods based on reference phantom measurements [4],[5]. In [7], we proposed and validated a linear least squares method, which includes both depth and frequency information in a single least-squares problem (as opposed to [5],[6]), thereby improving accuracy. Furthermore, the attenuation estimate is obtained through a closed-form equation, making it very cheap to compute and suitable for real-time attenuation estimation. In this paper, we extend this framework in [7] towards a multi-layer setting, in order to enable the estimation of spatial variations in the attenuation coefficient. We propose a linear model that incorporates all layers and their cumulative attenuation across the depth dimension in the medium. The resulting least-squares problem can again be solved in closed form, providing an estimate of the attenuation coefficient in each layer. The method is validated on simulated backscatter data.

Basic signal model:
For a plane wave, the magnitude of the backscattered signal spectrum (S) as a function of depth (z) and frequency (f) can be expressed as [7]

$$|S(f,z)| = G\,|P(f)|e^{-2\alpha f z},$$

where α denotes the attenuation coefficient, G denotes a gain calibration factor which accounts for how much energy is actually transmitted into the medium, and $P(f)$ represents the spectrum of the electrical signal used to excite the transducer along with the combined effect of electro-mechanical and mechano-electric coupling during the transfer from the electrical signal to an acoustic wave and vice-versa. Longitudinal plane-wave propagation is assumed, hence the effects due to diffraction (i.e. beam forming) and the frequency dependency of the backscatter are neglected.

Spatially varying signal model:
The above model can be extended for a medium with an arbitrary number of layers with different attenuation characteristics. We assume that the acoustic impedance between two adjacent layers is not very different so that the reflections and transmission losses at the interfaces can be neglected. Thus, for a medium with L distinct layers, the backscattered spectrum from a scatterer at depth z within the ith layer is given by

$$S(f,z) \quad = G\,P(f)e^{-2\alpha_1 f D_1}e^{-2\alpha_2 f D_2} \ldots e^{-2\alpha_{i-1} f D_{i-1}}e^{-2\alpha_i f\,(z-\sum_{j=1}^{i-1} D_j)} \quad , (1)$$

where $\alpha_1, \alpha_2 \ldots \alpha_i$ are the attenuation coefficients of the layers from 1 to i, and D_j is the thickness of the j^{th} layer which is assumed to be known. We are interested in estimating each of these attenuation coefficients. Equation (1) shows a nonlinear relationship of $\alpha_i s$ with respect to the backscattered spectrum. By using a logarithmic transformation and a suitable rearrangement of the terms, equation (1) can be expressed as a system of linear equations, as explained below.

Defining $Q(f,z) = \ln|S(f,z)| - \ln|P(f)|$, the backscattered data from each layer can be expressed as

$$Q(f,z) = \begin{cases} \ln|G| - 2a_1 fz & : D_0 < z \leq D_1 \\ \ln|G| - 2a_1 fD_1 - 2a_2 f(z - D_1) & : D_1 < z \leq D_2 \\ \vdots & \\ \ln|G| - 2a_1 fD_1 - 2a_2 fD_2 - \cdots - 2a_L f(z - \sum_{i=1}^{L-1} D_i) & : D_{L-1} < z \leq D_L \end{cases} \quad (2)$$

where $\ln$ denotes the natural logarithm.

Using a sliding window that slides over the z-axis at discrete positions, and setting $\boldsymbol{q}(f) = [Q(f, z_{1,1}), \ldots Q(f, z_{1,N_1}), \ldots, Q(f, z_{L,1}), \ldots, Q(f, z_{L,N_L})]^T$, where $z_{i,j}$ represents the jth window position in the ith layer, q = $[\ln|G|, \alpha_1, \alpha_2 \ldots \alpha_L]^T$, and

$$\boldsymbol{A}(f) = \begin{pmatrix} 1 & -2fz_{1,1} & 0 & 0 & \ldots & 0 \\ 1 & -2fz_{1,2} & 0 & 0 & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ldots & 0 \\ 1 & -2fz_{1,N} & 0 & 0 & \ldots & 0 \\ 1 & -2fD_1 & -2f(z_{2,1} - D_1) & 0 & \ldots & 0 \\ 1 & -2fD_1 & -2f(z_{2,2} - D_1) & 0 & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & -2fD_1 & -2fD_2 & -2fD_3 & \ldots & -2f(z_{L,N} - \sum_{i=1}^{L-1} D_i) \end{pmatrix},$$

the spatially varying signal model can be expressed as

$$\boldsymbol{q}(f) = \boldsymbol{A}(f)\ \boldsymbol{\theta}. \quad (3)$$

Note that one such set of equations can be generated for each frequency f. By vertically stacking these equations for all (discretized) frequencies within the relevant bandwidth, we obtain the complete system of linear equations

$$\boldsymbol{q} = \boldsymbol{A}\ \boldsymbol{\theta}. \quad (4)$$

Method of Estimation:
When the measured spectrum $\tilde{S}(f,z)$ is used in the calculation of $Q(f,z)$, the term $\boldsymbol{q}$ in equation (4) is replaced with $\tilde{\boldsymbol{q}}$. The estimation of $\boldsymbol{\theta}$ can now be posed as a linear least squares problem:

$$\widehat{\boldsymbol{\theta}} = \arg\min_{\theta} \|\tilde{\boldsymbol{q}} - \boldsymbol{A}\ \boldsymbol{\theta}\|_2^2 \quad (5)$$

This results in a closed form expression for the estimate of $\boldsymbol{\theta}$**,** given by

$$\widehat{\boldsymbol{\theta}} = (\boldsymbol{A}^T\boldsymbol{A})^{-1}\boldsymbol{A}^T\tilde{\boldsymbol{q}}. \quad (6)$$

Note that the matrix $(\boldsymbol{A}^T\boldsymbol{A})^{-1}\boldsymbol{A}^T$ can be precomputed as it is independent of the measured spectrum. Thus, the estimates can be obtained by a simple matrix-vector multiplication as shown in equation (6).

Experiments and Results: The validation of the proposed method was conducted with simulated data. The transmitted pulse had a center frequency of 2.25 MHz; a speed of sound of 1500 m/s; and a Gaussian shaped spectrum. The simulated medium had a thickness of 40 mm, in which the first 20 mm has an attenuation coefficient of α =0.8 dB/cm/MHz and the next 20 mm region α =0.4dB/cm/MHz. Approximately 530 scatterers per mm were used in the medium. 1500 RF lines were generated, each with independently and randomly drawn positions of the scatterers. The estimation was performed by randomly selecting N RF lines from this set without repetition, where N is varied from 1 to 50. During the estimation, the window-length chosen was 8mm, window overlap was 2mm and the usable bandwidth was 10 dB below the peak of the spectrum. The windows were chosen in such a way that the same window does not occupy two adjacent layers.

The plots in Fig. 1(a) and 1(b) show the relative error of estimation as a function of the number of RF lines for an insonification at either side of the medium. As the number of RF lines increases, the relative error decreases and stays almost constant. In both cases of insonification, the relative error is smaller for the layer with $\alpha = 0.8$ dB/cm/MHz. This is because the higher the value of the attenuation coefficient, the more pronounced the effect on the spectrum.

Conclusion:
A linear least squares method for estimating the spatial variation of the attenuation coefficient is proposed with preliminary validation for a two-layered case using simulated data. The method is computationally fast as the estimates can be obtained by a simple matrix-vector multiplication. Hence, it can be used for time gain compensation in real-time imaging applications. Furthermore, we have demonstrated on simulated data that the method provides accurate estimates.

Future scope:
In this paper, the estimation of the attenuation coefficient was performed by assuming a simplified model as given in equation (1). In this model, the assumption is made that there is no impedance mismatch between adjacent layers. If there are impedance mismatches, a component of the transmitted ultrasound from one layer to another will be reflected back, which is not yet accounted for in the current model. The model also assumes longitudinal plane wave propagation. However in actual setups diffraction occurs. Future work will focus on including such effects in the model, and relaxing other assumptions such as the assumed prior knowledge of the thickness of each layer.

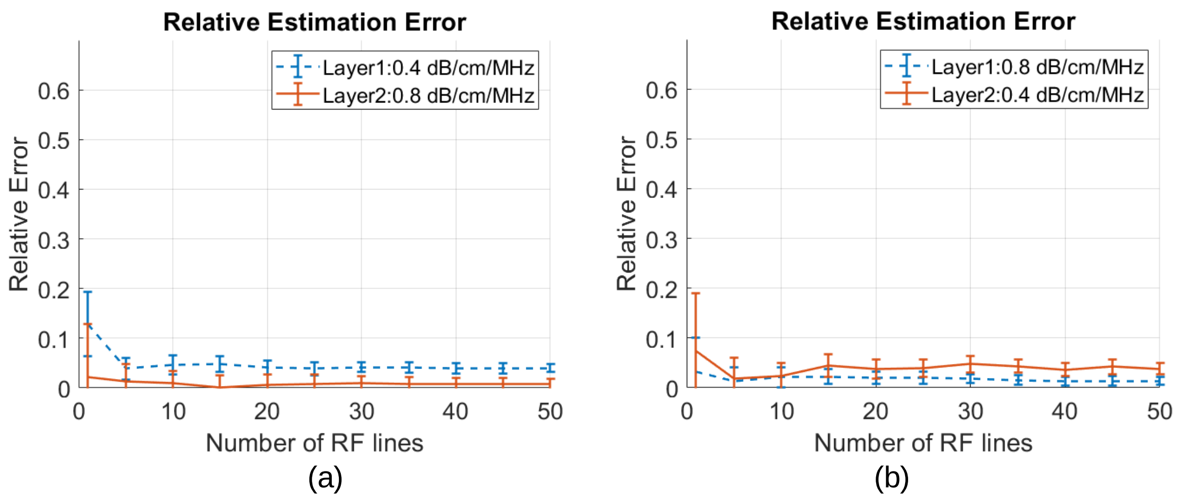

Figure 1: The relative error of the estimation of the attenuation coefficient as a function of the number of RF lines for a 2-layered medium with layer 1 with $\alpha = 0.4$ dB/cm/MHz and layer 2 with $\alpha = 0.8$ dB/cm/MHz, using simulated data. Subfigure (a) shows the relative estimation error for each layer, when the insonification is done on the side of layer 1, and subfigure (b) shows the case when the insonification is reversed.

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