# DISTRIBUTED LCMV BEAMFORMING IN WIRELESS SENSOR NETWORKS WITH NODE-SPECIFIC DESIRED SIGNALS

Alexander Bertrand\*, Marc Moonen

Katholieke Universiteit Leuven - Dept. ESAT Kasteelpark Arenberg 10, B-3001 Leuven, Belgium E-mail: alexander.bertrand@esat.kuleuven.be; marc.moonen@esat.kuleuven.be

# ABSTRACT

We consider distributed linearly constrained minimum variance (LCMV) beamforming in a wireless sensor network. Each node computes an LCMV beamformer with node-specific constraints, based on all sensor signals available in the network. A node has a local sensor array, and compresses its sensor signals to a signal with fewer channels, which is then shared with other nodes in the network. The compression rate depends inversely on the total number of linear constraints. Even though a significant compression is obtained, each node is able to generate the same outputs as a centralized LCMV beamformer, as if all sensor signals are available to every node. Since the distributed LCMV algorithm exploits a similar parametrization as previously developed distributed unconstrained MMSE signal estimation algorithms, it has similar dynamics and convergence properties. We provide simulation results to demonstrate the optimality and convergence of the algorithm.

*Index Terms*— Wireless sensor networks, random arrays, beamforming, distributed estimation

## 1. INTRODUCTION

Traditional sensor arrays for spatial filtering or beamforming contain a limited number of wired sensors, where all sensor observations are gathered in a central processor. Due to the relatively small size of the array, the spatial field is only sampled locally, and the target source(s) are often at a relatively large distance from the array, which yields sensor signals with low SNR. Recently, there has been a growing interest in distributed beamforming or signal estimation in wireless sensor networks (WSN's), where multiple sensor nodes are spread over the environment [1-3]. Each node consists of a small sensor array, a signal processing unit, and a wireless link with other nodes. The nodes then exchange compressed signal observations, and they cooperate in a distributed fashion to estimate the desired signal, based on all observations in the network. The advantages are that more sensors can be used, that the sensors physically cover a wider area, and that there is a higher probability that a node is close to the target source, yielding higher SNR signals to start with.

In some particular cases, it may be required to let each node estimate a different desired signal, or a locally observed version of a common signal, e.g. if it is followed by a localization task. This makes the estimation problem node-specific. Distributed node-specific signal enhancement was first considered in a 2-node network, in the context of binaural hearing aids where it is important to preserve the spatial cues of the desired signals at both ears [4]. This technique relies on the speech-distortion-weighted multi-channel Wiener filter (SDW-MWF), and was referred to as distributed MWF (DB-MWF). In [5], distributed minimum variance distortionless response (DB-MVDR) beamforming was introduced for a similar binaural setting, which is a special case of the former<sup>1</sup>. Both techniques assume a single target source. In [1], a distributed adaptive node-specific signal estimation (DANSE) algorithm is described for fully connected WSN's, which generalizes DB-MWF to any number of nodes and multiple target sources.

Since the 2-node DB-MVDR beamforming in [5] is a special case of DB-MWF (for a single desired source), it is also implicitly covered by the DANSE framework. Although this link between DB-MVDR beamforming and DANSE is not obvious at first sight, it is an important observation since it implies that all results and extensions of DANSE also apply to DB-MVDR beamforming, i.e. procedures for multiple nodes, multiple sources [1], simply connected topologies [2], and robust implementations [3]. Furthermore, by exploiting the existing knowledge on the general DANSE framework, it is possible to generalize DB-MVDR to distributed linearly constrained minimum variance (LCMV) beamforming [7], allowing multiple linear constraints, which is the main contribution of this paper.

LCMV-beamforming is a well-known sensor array technique for noise reduction [7] where the goal is to minimize the output power of a multi-channel filter, under a set of linear constraints, e.g. to preserve desired source signals and (fully or partially) cancel interferers. It is noted that MVDR beamforming is a special case of LCMV beamforming [7]. In this paper, we will explain how LCMV beamforming can be performed in a distributed fashion in a WSN with any number of nodes and any number of source signals. We consider a blind approach that operates without knowledge on the array geometry or positions of the sources. However, this means that our approach is limited to scenarios that lend themselves to blind subspace estimation of desired sources and interferers. This is for example possible in speech enhancement, where both subspaces can be tracked based on non-stationarity and on-off behavior of the desired source(s) [3, 5, 8]. We also allow that the nodes solve nodespecific LCMV problems, i.e. with different linear constraints. For example, a desired source for one node may be an interfering source for another node. We will refer to this algorithm as linearly con-

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<sup>&</sup>lt;sup>1</sup>It is in fact a limit case where the trade-off parameter  $\mu \to 0$ . When using a rank-1 model (in the case of a single desired source), setting  $\mu = 0$  in SDW-MWF gives the same formula as MVDR [6].

strained DANSE (LC-DANSE), to emphasize the close relation with the DANSE algorithm in [1]. For the sake of an easy exposition, we only consider the case of fully connected networks. However, since LC-DANSE has similar dynamics and parametrizations as DANSE, all aforementioned extensions of DANSE also hold for LC-DANSE.

#### 2. CENTRALIZED BLIND LCMV BEAMFORMING

Consider a network with J sensor nodes and  $\mathcal{J} = \{1, ..., J\}$ . Node k collects observations from a complex-valued<sup>2</sup>  $M_k$ -channel sensor signal  $\mathbf{y}_k[t]$ , where t is the time index which will be omitted in the sequel. All  $\mathbf{y}_k$ 's are stacked in an M-channel signal  $\mathbf{y}$  with  $M = \sum_{k \in \mathcal{J}} M_k$ . We assume that there are K relevant<sup>3</sup> spatial point sources, such that  $\mathbf{y}$  is generated by the following linear model

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n} \tag{1}$$

where s is a stacked signal vector containing K relevant source signals, H is an  $M \times K$  steering matrix, and n is a noise component.

First, we consider centralized LCMV beamforming, so we assume that a node k has access to all channels of y. Let  $\mathcal{I}_k^d$  denote the set of indices that correspond to the  $N_k$  desired sources from s that node k wants to preserve in its node-specific estimation. The other  $P_k = K - N_k$  sources from s are assumed to be interferers, and their indices define the set  $\mathcal{I}_k^i$ . Similar to [8], the goal for node k is to estimate the mixture of the  $N_k$  desired signals from s as they impinge on one of node k's sensors, referred to as the reference sensor (assume w.l.o.g. that this is the first sensor, i.e.  $y_{k1}$ ).

It is noted that we do not necessarily intend to demix these sources, since this would require to blindly estimate the steering vector of each source separately, which is often difficult or impossible. For example, in the case of speech enhancement, one needs a voice activity detector (VAD) to estimate the speech subspace [1, 3, 8]. In a multiple speaker scenario, to estimate the steering vectors of each speaker separately, the VAD must be able to distinguish between different speakers (e.g. as in [9]). However, common VAD's are triggered by any (nearby) speakers, and therefore only the joint subspace can be identified. Let  $\mathbf{Q}_k^d$  denote the  $M \times N_k$  matrix with its columns defining an orthogonal basis for the desired subspace spanned by the columns of **H** with indices in  $\mathcal{I}_k^d$ . Similarly, let  $\mathbf{Q}_k^i$ denote the  $M \times P_k$  matrix containing an orthogonal basis for the interferer subspace corresponding to  $\mathcal{I}_k^i$ . In the sequel, we assume that both  $\mathbf{Q}_k^d$  and  $\mathbf{Q}_k^i$  can be blindly estimated from the sensor signals y (e.g. with techniques from [8]).

Node k will apply a linear M-dimensional estimator  $\mathbf{w}_k$  to the M-channel signal  $\mathbf{y}$  to compute the signal  $d_k = \mathbf{w}_k^H \mathbf{y}$  where H denotes the conjugate transpose operator. To this end, it will choose the  $\mathbf{w}_k$  that minimizes the variance of  $d_k$ , while preserving the desired signals in  $\mathcal{I}_k^d$ . If required, other constraints can be added, e.g. to (fully or partially) block the interferers in  $\mathcal{I}_k^i$ . More specifically, node k solves the following centralized LCMV problem:

$$\min_{\mathbf{w}_k} \|\mathbf{w}_k^H \mathbf{y}\|^2 \tag{2}$$

s.t.

$$\mathbf{Q}_k^H \mathbf{w}_k = \mathbf{f}_k \tag{3}$$

with

$$\mathbf{Q}_{k} = \begin{bmatrix} \mathbf{Q}_{k}^{d} & \mathbf{Q}_{k}^{i} \end{bmatrix}$$
(4)

$$\mathbf{f}_{k} = \begin{bmatrix} \mathbf{q}_{k}^{d}(1) \\ \epsilon \mathbf{q}_{k}^{i}(1) \end{bmatrix}$$
(5)

where  $\mathbf{q}_{k}^{d}(1)$  and  $\mathbf{q}_{k}^{i}(1)$  denote the first column of  $\mathbf{Q}_{k}^{dH}$  and  $\mathbf{Q}_{k}^{iH}$  respectively (corresponding to the reference sensor of node k), and where  $\epsilon$  is a user-defined gain<sup>4</sup>. The solution of this problem is given by [7]:

$$\hat{\mathbf{w}}_{k} = \mathbf{R}_{yy}^{-1} \mathbf{Q}_{k} \left( \mathbf{Q}_{k}^{H} \mathbf{R}_{yy}^{-1} \mathbf{Q}_{k} \right)^{-1} \mathbf{f}_{k}$$
(6)

with  $\mathbf{R}_{yy} = E\{\mathbf{yy}^H\}$  where  $E\{.\}$  denotes the expected value operator. It can be shown [8] that the signal components of  $\mathbf{s}$  in the output  $\hat{d}_k = \hat{\mathbf{w}}_k^H \mathbf{y}$ , are equal to the signals as they impinge on the reference sensor (except for some scaling by  $\epsilon$  for the interfererers), i.e.

$$\hat{d}_k = \sum_{l \in \mathcal{I}^d} h_{1l} s_l + \epsilon \sum_{l \in \mathcal{I}^i} h_{1l} s_l + \mathbf{V}_k \mathbf{n}$$
(7)

with  $h_{ij}$  denoting the entry in the *i*-th row and *j*-th column of **H**, and with

$$\mathbf{V}_{k} = \mathbf{f}_{k}^{H} \left( \mathbf{Q}_{k}^{H} \mathbf{R}_{nn}^{-1} \mathbf{Q}_{k} \right)^{-1} \mathbf{Q}_{k}^{H}$$
(8)

where  $\mathbf{R}_{nn} = E\{\mathbf{nn}^H\}$ . It is noted that this procedure yields a distortionless response, which is not the case in SDW-MWF based beamforming techniques [4]. However, the constraints that enforce this distortionless response remove some degrees of freedom, yielding less noise reduction in the residual  $\mathbf{V}_k \mathbf{n}$ .

#### 3. LINEARLY CONSTRAINED DANSE (LC-DANSE)

In this section, we propose a distributed adaptive node-specific LCMV beamforming algorithm that obtains the centralized estimates (7),  $\forall k \in \mathcal{J}$ , and where nodes exchange linearly compressed signal observations. We will refer to this algorithm as linearly constrained DANSE (LC-DANSE), as it is based on the DANSE algorithm that was originally proposed for linear MMSE signal estimation [1]. For the sake of an easy exposition, we describe the algorithm for a fully connected network, but all results can be extended to simply connected networks, similar to [2].

The iterative nature of the algorithm may suggest that the same data is re-estimated and transmitted multiple times. However, in practice, iterations can be spread out over different data blocks (see remark below). In the case where there are K relevant point sources, the nodes will exchange K-channel signal observations, yielding a compression with a factor of  $M_k/K$  at node k, where we assume<sup>5</sup> that  $M_k > K$ .

#### 3.1. The LC-DANSE algorithm

First, we define K - 1 auxiliary estimation problems at node k, which are basically the same as (2)-(3) but with different choices for  $\mathbf{f}_k$ . This means that node k computes K different beamformer outputs  $\mathbf{d}_k = \mathbf{W}_k^H \mathbf{y}$ , defined by an  $M \times K$  linear estimator  $\mathbf{W}_k$  that solves

$$\min_{\mathbf{W}_k} \|\mathbf{W}_k^H \mathbf{y}\|^2 \tag{9}$$

<sup>&</sup>lt;sup>2</sup>We assume that all signals are complex valued to incorporate frequency domain description, e.g. in the short-time Fourier transform (STFT) domain.

<sup>&</sup>lt;sup>3</sup>We consider a point source as relevant, if there is at least one node that uses this source in the linear constraints of its estimation problem, as explained later.

<sup>&</sup>lt;sup>4</sup>Usually  $\epsilon = 0$  to fully cancel the interferers. However in some cases it may be important to retain some residual noise, e.g. for hearing aid users in traffic situations.

<sup>&</sup>lt;sup>5</sup>In the fully connected case, LC-DANSE only has a benefit if  $M_k > K$ . In the case of a simply connected topology (see [2]), there is still a benefit compared to the scenario where all signals are relayed, even if  $M_k < K$ .

$$\mathbf{Q}_k^H \mathbf{W}_k = \mathbf{F}_k \tag{10}$$

where  $\mathbf{F}_k$  is chosen as a full rank  $K \times K$  matrix. The last K - 1 columns of  $\mathbf{F}_k$  may be filled with constraints that define other interesting estimation problems for node k that use the same partitioning of the two subspaces  $\mathbf{Q}_k^d$  and  $\mathbf{Q}_k^i$ . In the sequel, we assume that  $\mathbf{F}_k$  has the form

$$\mathbf{F}_{k} = \begin{bmatrix} \alpha_{1} \mathbf{q}_{k}^{d}(m_{1}) & \alpha_{2} \mathbf{q}_{k}^{d}(m_{2}) & \dots & \alpha_{K} \mathbf{q}_{k}^{d}(m_{K}) \\ \epsilon_{1} \mathbf{q}_{k}^{i}(n_{1}) & \epsilon_{2} \mathbf{q}_{k}^{i}(n_{2}) & \dots & \epsilon_{K} \mathbf{q}_{k}^{i}(n_{K}) \end{bmatrix}$$
(11)

where  $m_j$ ,  $n_j \in \{1, ..., M_k\}$  and where  $\alpha_j, \epsilon_j \in \mathbb{C}$  are chosen such that  $\mathbf{F}_k$  is full rank. This incorporates all estimation problems that use the same subspace partitioning<sup>6</sup>. The solution of (9)-(10) is

$$\hat{\mathbf{W}}_{k} = \mathbf{R}_{yy}^{-1} \mathbf{Q}_{k} \left( \mathbf{Q}_{k}^{H} \mathbf{R}_{yy}^{-1} \mathbf{Q}_{k} \right)^{-1} \mathbf{F}_{k} .$$
(12)

The reason for adding these auxiliary estimation problems, is to obtain an estimator  $\mathbf{W}_k$  that captures the full *K*-dimensional signal subspace defined by the channels of s.

In the LC-DANSE algorithm,  $\mathbf{y}_k$  is linearly compressed to a K-channel signal  $\mathbf{z}_k$  (the compression rule will be defined later), which is then broadcast to the remaining J - 1 nodes. We define the (K(J-1))-channel signal  $\mathbf{z}_{-k} = [\mathbf{z}_1^T \dots \mathbf{z}_{k-1}^T \mathbf{z}_{k+1}^T \dots \mathbf{z}_J^T]^T$ . Node k then collects observations of its own sensor signals in  $\mathbf{y}_k$  and the channels of  $\mathbf{z}_{-k}$  obtained from the other nodes in the network. Similar to the centralized LCMV approach, node k can then compute the  $(M_k + K(J-1))$ -channel LCMV beamformer  $\mathbf{U}_k$  with respect to these input signals, i.e. the solution of

$$\min_{\mathbf{U}_{k}} \|\mathbf{U}_{k}^{H}\tilde{\mathbf{y}}_{k}\|^{2}$$
(13)

s.t.

$$\widetilde{\mathbf{Q}}_{k}^{H}\mathbf{U}_{k} = \widetilde{\mathbf{F}}_{k} \tag{14}$$

where

$$\tilde{\mathbf{y}}_k = \begin{bmatrix} \mathbf{y}_k \\ \mathbf{z}_{-k} \end{bmatrix}$$
(15)

and with  $\hat{\mathbf{Q}}_k$  denoting the equivalent to  $\mathbf{Q}_k$ , but now with respect to the modified steering vectors corresponding to node k's input signals, i.e.  $\tilde{\mathbf{y}}_k$ . These will be linearly compressed versions of the steering vectors in **H**, due to the linear compression rules that generate the  $\mathbf{z}_k$ . Since we assumed that the subspaces spanned by the steering vectors can be estimated blindly from the input signals,  $\tilde{\mathbf{Q}}_k$ can be computed.  $\tilde{\mathbf{F}}_k$  is constructed similarly to (11), but now with respect to the columns of  $\tilde{\mathbf{Q}}_k^H$  instead of  $\mathbf{Q}_k^H$ .

The problem (13)-(14) is equivalent to the centralized LCMV problem described in Section 2 (but with fewer signals), and its solution can be computed in exactly the same way.

We now define the partitioning

$$\mathbf{U}_{k} = \begin{bmatrix} \mathbf{W}_{kk}^{T} | \mathbf{G}_{k,-k}^{T} \end{bmatrix}^{T}$$
(16)  
= 
$$\begin{bmatrix} \mathbf{W}_{kk}^{T} | \mathbf{G}_{k1}^{T} | \dots | \mathbf{G}_{k,k-1}^{T} | \mathbf{G}_{k,k+1}^{T} | \dots | \mathbf{G}_{kJ}^{T} \end{bmatrix}^{T}$$
(17)

where  $\mathbf{W}_{kk}$  contains the first  $M_k$  rows of  $\mathbf{U}_k$  (which are applied to node k's own sensor signals  $\mathbf{y}_k$ ) and where  $\mathbf{G}_{kq}$  is the part of  $\mathbf{U}_k$ 



**Fig. 1.** The LC-DANSE scheme with 3 nodes (J = 3). Each node k computes an LCMV beamformer using its own  $M_k$ -channel sensor signal observations, and 2 K-channel signals broadcast by the other two nodes.

that is applied to the K-channel signal  $\mathbf{z}_q$  obtained from node q. We can now also define the compression rule to generate the broadcast signal  $\mathbf{z}_k$  as

$$\mathbf{z}_k = \mathbf{W}_{kk}^H \mathbf{y}_k \;. \tag{18}$$

A schematic illustration of this scheme is shown in Fig. 1, for a network with J = 3 nodes. It is noted that  $\mathbf{W}_{kk}$  both acts as a compressor and as a part of the estimator  $\mathbf{W}_k$ . Based on Fig. 1, it can be seen that the parametrization of  $\mathbf{W}_k$  effectively applied at node k, to generate  $\mathbf{d}_k = \mathbf{W}_k^H \mathbf{y}$ , is then

$$\mathbf{W}_{k} = \begin{bmatrix} \mathbf{W}_{11}\mathbf{G}_{k1} \\ \vdots \\ \mathbf{W}_{JJ}\mathbf{G}_{kJ} \end{bmatrix}$$
(19)

where we assume that  $\mathbf{G}_{kk} = \mathbf{I}_K$  with  $\mathbf{I}_K$  denoting the  $K \times K$  identity matrix. This is exactly the same parametrization as used in the DANSE algorithm [1]. If we define the partitioning  $\mathbf{W}_k = \begin{bmatrix} \mathbf{W}_{k1}^T \dots \mathbf{W}_{kJ}^T \end{bmatrix}^T$ , where  $\mathbf{W}_{kq}$  is the part of  $\mathbf{W}_k$  that is applied to the sensor signals of node q, i.e.  $\mathbf{y}_q$ , then (19) is equivalent to

$$\mathbf{W}_{kq} = \mathbf{W}_{qq} \mathbf{G}_{kq}, \, \forall \, k, q \in \mathcal{J} \,. \tag{20}$$

Expression (19) defines a solution space for all  $\mathbf{W}_k$ ,  $k \in \mathcal{J}$ , simultaneously, where node k can only control the parameters  $\mathbf{W}_{kk}$  and  $\mathbf{G}_{k,-k}$ . The following theorem explains how this parametrization is still able to provide the optimal LCMV solution in each node.

**Theorem 3.1.** If  $\mathbf{F}_k$  in (11) is full rank,  $\forall k \in \mathcal{J}$ , then the optimal estimators  $\hat{\mathbf{W}}_k$ ,  $\forall k \in \mathcal{J}$ , given in (12) are in the solution space defined by parametrization (19).

*Proof.* Since the columns of  $\mathbf{Q}_k$  span the same subspace as the columns of  $\mathbf{H}$ , and since  $\mathbf{F}_k$  is full rank, there exists a full rank  $K \times K$  matrix  $\mathbf{A}_k$  such that

$$\mathbf{Q}_{k} \left( \mathbf{Q}_{k}^{H} \mathbf{R}_{yy}^{-1} \mathbf{Q}_{k} \right)^{-1} \mathbf{F}_{k} = \mathbf{H} \mathbf{A}_{k} .$$
 (21)

Substituting (21) in (12) shows that

$$\forall \, k, q \in \mathcal{J} : \hat{\mathbf{W}}_k = \hat{\mathbf{W}}_q \mathbf{A}_{kq} \tag{22}$$

with  $\mathbf{A}_{kq} = \mathbf{A}_q^{-1}\mathbf{A}_k$ . The theorem is proven by comparing (22) with (20), and by setting  $\mathbf{G}_{kq} = \mathbf{A}_{kq}, \forall q \in \mathcal{J}$ .

<sup>&</sup>lt;sup>6</sup>The last K - 1 columns of  $\mathbf{F}_k$  may also be filled with random entries. This often yields a better conditioned system, but then the equivalence of the solutions of LC-DANSE and the centralized LCMV problems, as described in Theorem 3.2, only holds for the first column of  $\mathbf{W}_k$ . This is however not a problem, since the other estimator columns are auxiliary.

The LC-DANSE algorithm iteratively updates the parameters in (19), by letting each node k compute (13)-(14),  $\forall k \in \mathcal{J}$ , in a sequential round robin fashion:

- 1. Initialize  $i \leftarrow 0, k \leftarrow 1$ , and initialize all  $\mathbf{U}_q^0, \forall q \in \mathcal{J}$ , with random entries.
- 2. Update  $\hat{\mathbf{Q}}_k$  and  $\hat{\mathbf{F}}_k$  by estimating the (orthogonalized) desired and interferer subspace with respect to the new inputs.
- Update U<sup>i</sup><sub>k</sub> to U<sup>i+1</sup><sub>k</sub> according to the solution of (13)-(14), while the other nodes do not perform any updates, i.e. U<sup>i+1</sup><sub>q</sub> = U<sup>i</sup><sub>q</sub>, ∀ q ∈ J \{k}.
- 4.  $i \leftarrow i + 1$  and  $k \leftarrow (k \mod J) + 1$ .
- 5. Return to step 2.

**Remark:** The iterative nature of the LC-DANSE algorithm may suggest that the same sensor signal observations are compressed and broadcast multiple times, i.e. once after every iteration. However, as mentioned earlier, iterations can be spread over time in practice. This means that there is no iterative estimation over the data blocks, only over the local fusion rules. In other words, if  $\mathbf{W}_{kk}^{i+1}$  is updated to  $\mathbf{W}_{kk}^{i+1}$  at time  $t_0$ , this updated version is only used to produce samples of  $z_k[t]$  for which  $t > t_0$ , while previous observations for  $t \leq t_0$ , are neither recompressed nor retransmitted. Effectively, each sensor signal observation is compressed and transmitted only once.

#### 3.2. Convergence and optimality of LC-DANSE

The following theorem guarantees convergence and optimality of LC-DANSE:

**Theorem 3.2.** If  $\mathbf{F}_k$  in (11) is full rank,  $\forall k \in \mathcal{J}$ , then all parameters of the LC-DANSE algorithm converge. Furthermore, if  $i \to \infty$ , the output signal  $d_k^i$  is equal to the output signal of the centralized algorithm defined in (7),  $\forall k \in \mathcal{J}$ , and the estimator  $\mathbf{W}_k^i$  parametrized by (19) is equal to  $\hat{\mathbf{W}}_k$ ,  $\forall k \in \mathcal{J}$ .

We will not formally prove this theorem here due to space constraints, but we give a brief intuition instead. The algorithm exploits the fact that an update of node k is also optimal for any other node q, if the latter is allowed to perform an optimal  $K \times K$  transformation on each input signal  $\mathbf{z}_{ql}$ ,  $\forall l \in \mathcal{J}$ . This follows from the fact that the LCMV solutions (12) all share the same K-dimensional subspace,  $\forall k \in \mathcal{J}$ . Therefore, although the updates of each node are 'selfish' in the sense that they only take their own estimation problem into account, the nodes have an implicit cooperative behavior, which yields convergence and optimality.

#### 4. SIMULATION

We simulated a toy scenario with K = 3 relevant white Gaussian point sources with unit variance, J = 10 nodes, each having  $M_k =$ 6 sensors (M = 60). The steering vectors to each sensor are chosen randomly from a zero-mean uniform distribution in [-0.5, 0.5]. The sensor noise power is 25% of the power of the relevant sources, and spatially uncorrelated. Each node selects randomly which of the 3 relevant sources are assumed to be targets or interferers.

The upper plot in Fig. 2 shows the power of the distributed LC-DANSE output for node 1, compared to the output power of the centralized LCMV beamformer, over the different iterations of the algorithm. In each iteration, a different node performs an update, starting with node 1 (round robin). It is observed that, when each node has updated twice, the LC-DANSE algorithm reaches the same



**Fig. 2.** Output power of LC-DANSE at node 1 (above) and the squared error  $\sum_{k \in \mathcal{J}} \|\mathbf{W}_k - \hat{\mathbf{W}}_k\|_F^2$  (below) vs. the number of iterations.

performance as the centralized algorithm in every node. In some iterations, the variance is lower than in the centralized solution, which is possible due to unsatisfied constraints at node 1 after updates at other nodes. The lower plot shows  $\sum_{k \in \mathcal{J}} ||\mathbf{W}_k - \hat{\mathbf{W}}_k||_F^2$  ( $||.||_F$  is a Frobenius norm) over the different iterations, i.e. the squared distance between the LC-DANSE estimators  $\mathbf{W}_k$ , parametrized according to (19) and the optimal solution  $\hat{\mathbf{W}}_k$  over all nodes.

### 5. CONCLUSIONS

In this paper, we have introduced a distributed adaptive node-specific LCMV beamforming algorithm, referred to as LC-DANSE. The algorithm significantly compresses the sensor signal observations that are shared between nodes, but obtains the same node-specific LCMV beamformers as the centralized algorithm at each node. The algorithm is closely related to the DANSE algorithm for unconstrained linear MMSE signal estimation, and we pointed out that previously developed extensions for DANSE also hold for LC-DANSE. We provided a simulation result to show the effectiveness of our method.

#### 6. REFERENCES

- A. Bertrand and M. Moonen, "Distributed adaptive node-specific signal estimation in fully connected sensor networks – part I: sequential node updating," *IEEE Transactions on Signal Processing*, vol. 58, pp. 5277–5291, 2010.
- [2] —, "Distributed adaptive estimation of node-specific signals in wireless sensor networks with a tree topology," *Internal report Katholieke Universiteit Leuven*, SCD/SISTA, 2010.
- [3] —, "Robust distributed noise reduction in hearing aids with external acoustic sensor nodes," *EURASIP Journal on Advances in Signal Processing*, vol. 2009, Article ID 530435, 14 pages, 2009.
- [4] S. Doclo, T. van den Bogaert, M. Moonen, and J. Wouters, "Reduced-bandwidth and distributed MWF-based noise reduction algorithms for binaural hearing aids," *IEEE Trans. Audio, Speech and Language Processing*, vol. 17, pp. 38–51, Jan. 2009.
- [5] S. Markovich Golan, S. Gannot, and I. Cohen, "A reduced bandwidth binaural MVDR beamformer," in *Proc. of the International Workshop on Acoustic Echo and Noise Control (IWAENC)*, Tel-Aviv, Israel, Aug. 2010.
- [6] M. Souden, J. Benesty, and S. Affes, "On optimal frequency-domain multichannel linear filtering for noise reduction," *Audio, Speech, and Language Processing, IEEE Transactions on*, vol. 18, no. 2, pp. 260–276, 2010.
- [7] B. Van Veen and K. Buckley, "Beamforming: a versatile approach to spatial filtering," ASSP Magazine, IEEE, vol. 5, no. 2, pp. 4 –24, apr. 1988.
- [8] S. Markovich Golan, S. Gannot, and I. Cohen, "Subspace tracking of multiple sources and its application to speakers extraction," in *Proc. IEEE Int. Conf. Acoustics, Speech, and Signal Processing (ICASSP)*, Dallas, Texas USA, Mar. 2010, pp. 201–204.
- [9] A. Bertrand and M. Moonen, "Energy-based multi-speaker voice activity detection with an ad-hoc microphone array," in *Proc. IEEE Int. Conf. Acoustics, Speech, and Signal Processing (ICASSP)*, Dallas, Texas USA, March 2010, pp. 85–88.